Quantum compression relative to a set of measurements

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(with Michael M. Wolf and Lukas Rauber)

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From now on: Measurement = Set of effect operators

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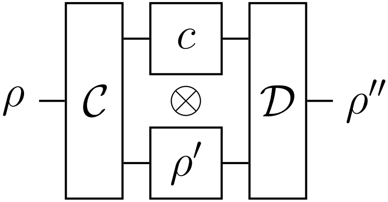

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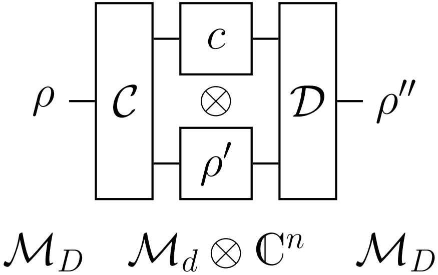

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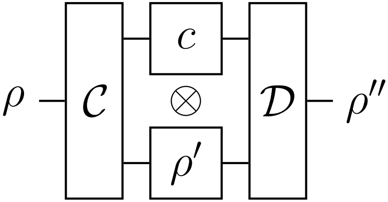

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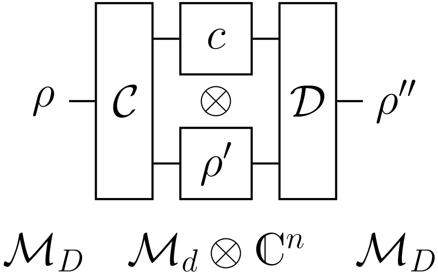

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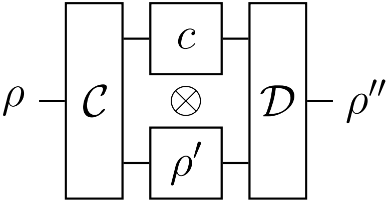

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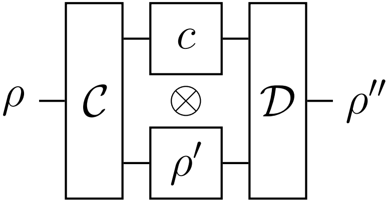

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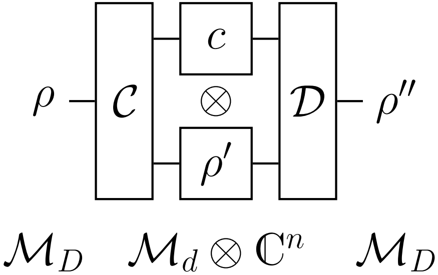

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- Aim: Find d as small as possible, n may be arbitrarily large
- This d is the compression dimension of $\mathcal{O}$

Stability of the compression dimension

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Theorem

Let d be the compression dimension for $\mathcal{O}$ and $\mathcal{O}$ be compact. Then there is an $\epsilon > 0$ such that for $d' < d$ and for every CPTP maps $\mathcal{C} : \mathcal{M}_D \to \mathcal{M}_{d'} \otimes \mathbb{C}^n$, $\mathcal{D} : \mathcal{M}_{d'} \otimes \mathbb{C}^n \to \mathcal{M}_D$, $n \in \mathbb{N}$, there are $\rho \in \mathcal{S}(\mathbb{C}^D)$ and $E \in \mathcal{O}$ such that

$$|\mathrm{Tr}\,(\rho E) - \mathrm{Tr}\,((\mathcal{D} \circ \mathcal{C})[\rho] E)\,| \geq \epsilon.$$

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Proof idea:

- Classical side information can be bounded, i.e. for every $\mathcal{T} = \mathcal{D} \circ \mathcal{C}$, there are $\mathcal{C}'$, $\mathcal{D}'$ with $n \leq D^4$ and $\mathcal{T} = \mathcal{D}' \circ \mathcal{C}'$.

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For the rest of the talk, we can restrict to the exact case.

Algebraic approach

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$$\mathrm{C}^*(\mathcal{O}, \mathbb{1}) \simeq \begin{bmatrix} \mathcal{M}_{d_1} & & \\ & \mathcal{M}_{d_2} & \\ & & \mathcal{M}_{d_3} \end{bmatrix}$$

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Theorem (Lower/Upper bound compression dimension)

Let d be the compression dimension of $\mathcal{O}$ and $\{ D_i \}$ be the dimensions of the matrix algebras occuring in the representation of $\mathrm{C}^*(\mathcal{O}, \mathbb{1})$. Then it holds that

$$\min_{i \in [s]} D_i \leq d \leq \max_{i \in [s]} D_i.$$

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Example (Generic case)

Let A, $B \in \mathcal{M}_D^{\mathrm{herm}}$ be generic. Then $\mathrm{C}^*(\{A, B\}) \simeq \mathcal{M}_D$ and $\mathcal{O} := \{\mathbb{1}, A, B\}$ is incompressible ($d = D$).

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Example (Two bipartite von Neumann measurements)

Let P, $Q \in \mathcal{E}(\mathbb{C}^D)$ be two orthogonal projections. Then, $\mathcal{O} := \{P, \mathbb{1} - P, Q, \mathbb{1} - Q\}$ has compression dimension at most 2.

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The latter example breaks down for more projections.

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- Repeat for every block from largest to smallest until interpolation map no longer exists

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- Corollary: The same holds if we allow multiple copies $\rho^{\otimes m}$ instead of ρ as input for $\mathcal{C}$.
- This cannot be proved using the algebraic approach.

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$$\mathrm{Tr}\left(\rho E\right) = \mathrm{Tr}\left(\mathcal{T}(\rho) E\right) = \mathrm{Tr}\left(\rho \mathcal{T}^*(E)\right) \qquad \forall \rho \in \mathcal{S}(\mathbb{C}^D), \forall E \in \mathcal{O}$$

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- Bézout's theorem: Not possible if E_1, E_2 give rise to an irreducible characteristic polynomial

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