Sampling mixed quantum states

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Joint work with Philippe Lamontagne, Serge Fehr and Louis Salvail

Sampling

Classical certification

`Source`

$$\downarrow$$

$$\begin{array}{ccccccccccccccc}
X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} & X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\
X_{16} & X_{17} & X_{18} & X_{19} & X_{20} & X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} & X_{30} \\
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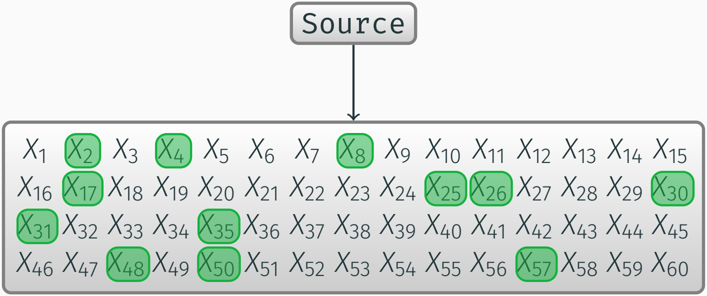
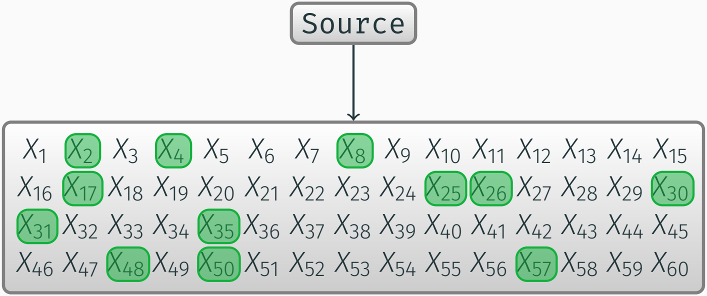
\end{array}$$

Classical certification

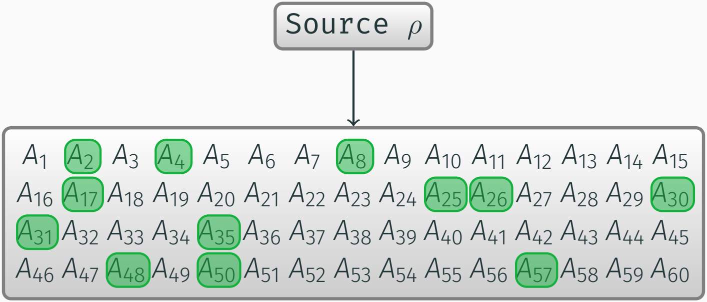




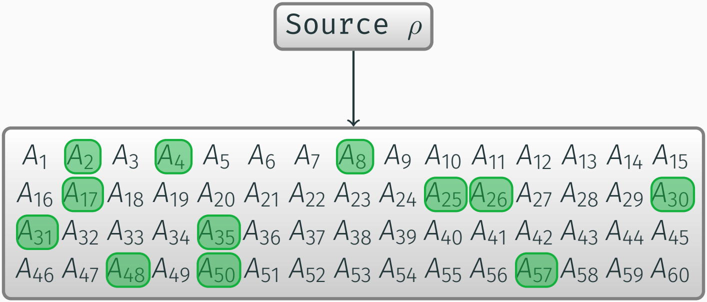
Suppose $X_i \in \{0, 1\}$. Then, sampling tells us:

- If we only see zeros in the sample $\overset{whp}{\Rightarrow}$ we should have at most δn 1's in the rest

Quantum certification



- Now, each A_i is a qubit
- Suppose we measure all the qubits in the sample in the computational basis, get all zeros
- What can we say about the state?



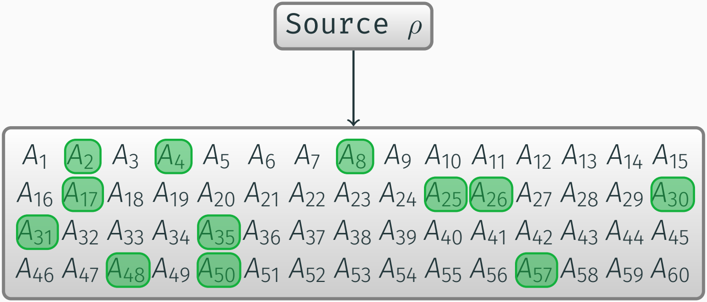
- We can define a *low-error subspace*

$$\mathcal{T}_\varepsilon := \mathsf{span}\left\{\left|x_1^{n-k}\right\rangle : x_1^{n-k} \text{ has at most } \varepsilon n \text{ 1's}\right\}$$

- Statement:

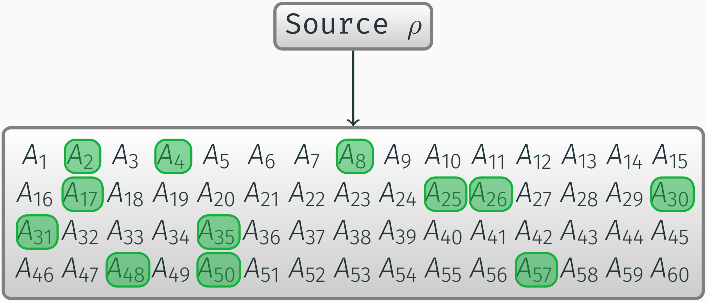
$$\mathsf{tr}\left[\rho_{\square}\Pi_{\mathcal{T}_\varepsilon}\right] \geqslant 1 - \mathsf{negl}$$

Quantum sampling



- Bouman and Fehr showed that any classical sampling procedure has a quantum analogue
- This works as long as we're certifying *pure* states
- What happens if we want to certify mixed states?

Certifying mixed states



- We now want to certify that most positions are in the mixed state φ
- We could measure sampled positions in the diagonal basis of φ, see if we get the right statistics
- This fails: a pure state with the right stats would pass the test

Certifying mixed states

- The task is impossible as it stands:
  - $\varphi^{\otimes n}$ is a mixture of pure states, each of which should fail the test
- Classically, the task also makes no sense
  - Looking at a bitstring, what probability distribution did it come from?
- It makes sense if we can ask for purifications:

$$\varphi_A \to |\varphi\rangle_{AR}$$

A mixed state certification protocol

A *interactive game* between two players: a Prover and a Verifier

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Verifier wants to certify that his state is close to $\varphi^{\otimes n}$. Prover wants to fool the verifier into thinking he has the right state even though it's not the case.

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- P. Prepare $|\varphi\rangle_{AR}^{\otimes n}$, send A^n to verifier.
- V. Choose a random sample, announce it to prover.
- P. Send R for each position in sample.
- V. Measure $\{|\varphi\rangle\langle\varphi|_{AR}, \mathbb{I} - |\varphi\rangle\langle\varphi|_{AR}\}$ for each joint system AR in sample.
- V. Accept if no errors, reject otherwise.

Is this protocol secure? What does it mean to be secure?

Applications

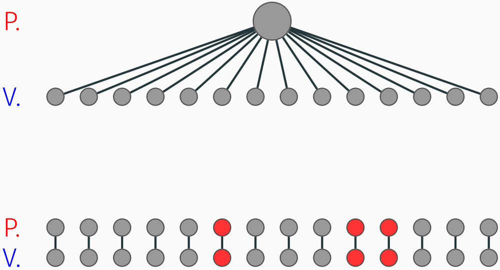
This has some applications in cryptography:

- Coin tossing: Alice prepares n EPR pairs, Bob certifies them, then they measure in the computational basis.
  - Caveat: we still get a few errors, no way to get rid of them $\Rightarrow$ we get a source of min-entropy arbitrarily close to n
- Preparing "magic states" for multiparty computation protocols

Defining security

How do we define security? Tempting definition:

- With high probability, the prover could produce purifications of the remaining systems with at most εn errors

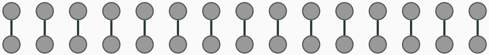


This definition doesn't work, because of *postselection attacks*

Postselection attack

P.

V.



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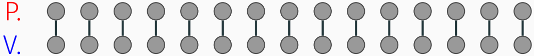
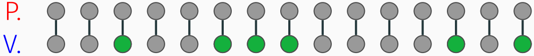


1. Learns sample

Postselection attack



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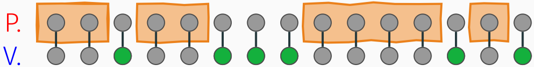

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Postselection attacks

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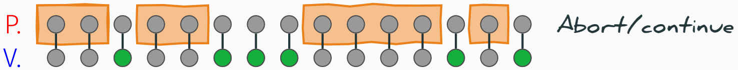

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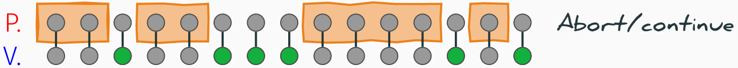

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Post-selection

Example

Prepare $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)^{\otimes n}$, measure positions outside of sample, abort if result $\neq |0\rangle^{\otimes n-k}$.

Resulting state always $|0\rangle^{\otimes n-k}$

What *can* the prover do?

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An “undetectable” attack

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- prepare a mixture/superposition of such states,
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ideal state

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Defining success

Definition (Soundness)

For any strategy for the prover, the output state ρ_{A^n} of the verifier is s.t.

RHS is "rough approximation" of LHS

$$\rho_{A^n} \leqslant p_n \cdot \psi_{A^n} + \sigma$$

where p_n is polynomial in n, ψ_{A^n} is part of an ideal state $|\psi\rangle_{A^n R^n E}$ and $\mathrm{tr}(\sigma) \leq \mathrm{negl}(n)$.

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Application

For any "bad event",

$$\Pr[\text{bad event} \mid \rho_{A^n}] \leq p_n \Pr[\text{bad event} \mid \psi_{A^n}] + \text{negl}(n)$$

Secure application if $\Pr[\text{bad event} \mid \psi_{A^n}]$ is negligible.

Main result

Our sampling protocol:

P. Prepare $|\varphi\rangle_{AR}^{\otimes n}$, send A^n to verifier.

V. Choose a random sample, announce it to prover.

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V. Measure $\{|\varphi\rangle\langle\varphi|_{AR}, \mathbb{I} - |\varphi\rangle\langle\varphi|_{AR}\}$ for each joint system AR in sample.

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Theorem (Main)

This protocol is sound.

Proof Tools and Sketch

Permutations and sampling are closely related

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Choosing a random subset of size k of a population

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22

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Sampling is "invariant under permutation".

Permutation invariance is a powerful tool

- Permutations are bijections of the form

$$\pi : \{1, \ldots, n\} \to \{1, \ldots, n\} \qquad \text{e.g. } \pi = \begin{pmatrix} 12345 \\ 53421 \end{pmatrix}$$

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- Permutation invariant operators are approximated by mixtures of i.i.d. operators [CKR09]

$$\rho = \pi \rho \pi^* \quad \forall \pi \implies \rho \leq p(n) \int \theta^{\otimes n} d\theta$$

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Rough sketch of the proof

- Protocol is invariant under permutation of verifier's registers **if** prover knows π.
- Randomly permute A^n, give π to prover.
- No loss of generality in assuming prover purifies choice of π.
- Equivalent to attack using permutation invariant $\rho_{A^n R^n}$
- $\Pr[\text{Attack} \mid \rho_{A^n R^n}] \leq \Pr[\text{Attack} \mid \int \theta_{AR}^{\otimes n}\, d\theta] \leq \text{negl}(n)$
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  easy
- Still need to unpermute verifier's output hard

Conclusion

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Open problems

- Certifying arbitrary reference states (vs $\varphi^{\otimes n}$)
- Do sampling in the more general sense of estimating the error rate.

Thank you!

The paper: `arXiv:1804.04730`