Vertices cannot be hidden from quantum spatial search for almost all random graphs

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A. Glos, A. Krawiec, R. Kukulski, and Z. Puchała, "Vertices cannot be hidden from quantum spatial search for almost all random graphs", *Quantum Information Processing*, vol. 17, pp. 81, 2018

There is a treasure...

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hidden in the castle.

http://myocn.net/treasure-on-earth-treasure-in-heaven/


slovakia.travel


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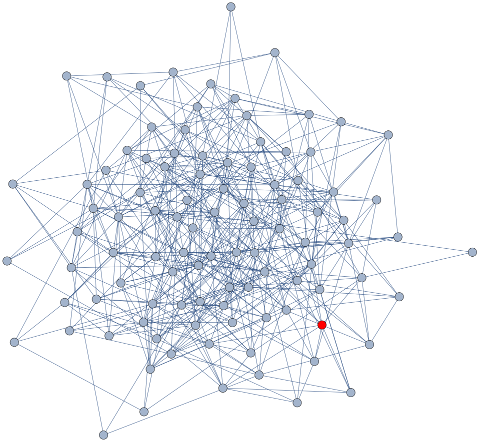
The problem



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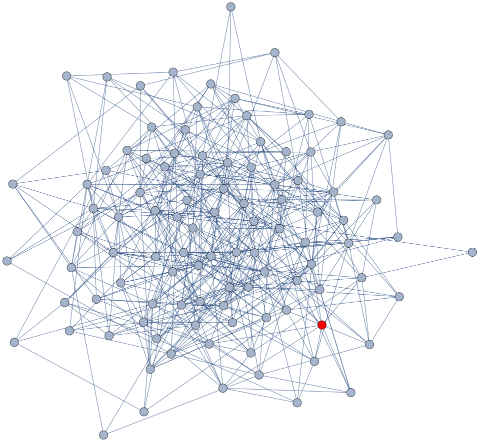

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$\Omega(n)$ at best

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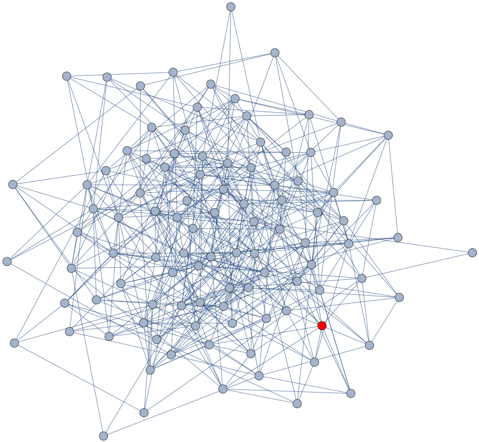

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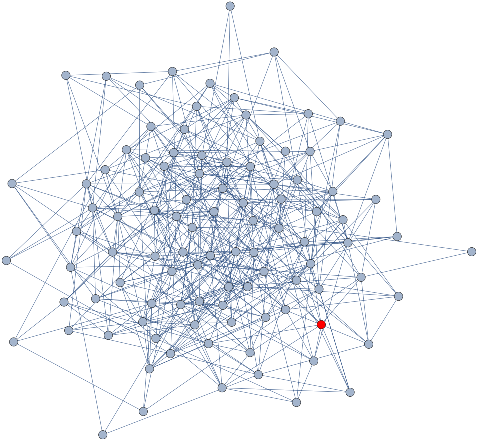

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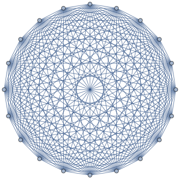
But..

For which graphs and vertices does it work?

Which graphs were analysed?



Complete graphs (Childs et al.) $\mathcal{O}(\sqrt{n})$

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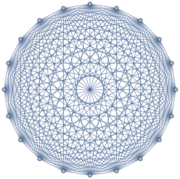

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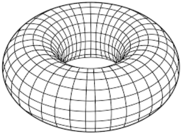

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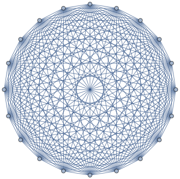

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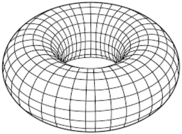

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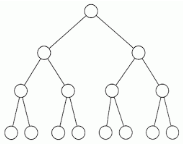

Full binary trees (Philipp et al.) $\mathcal{O}(n^{\frac{1}{2}+\delta})$ for $\delta > 0$

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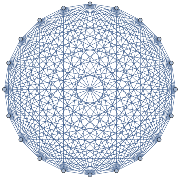

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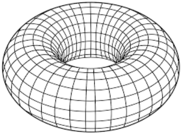

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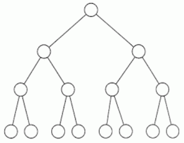

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Lack of generality...

Erdős-Rényi graph model

- $G = (V, E)$ - a simple undirected graph with vertex set $V = \{1, \ldots, n\}$ and edge set $E \subset V \times V$; $\mathcal{G}(n, p)$

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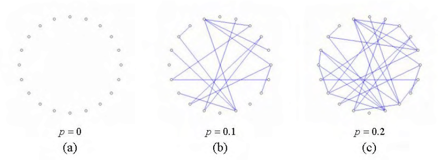

Figure: https://www.researchgate.net/publication/313854183_WISDOM-II

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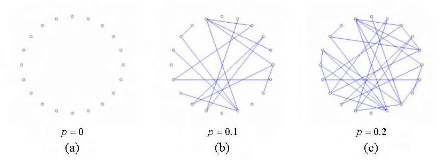

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- connectivity threshold

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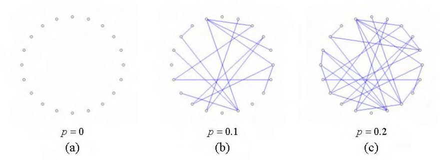

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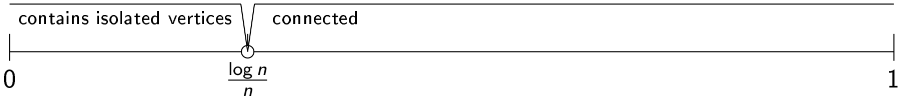

Result of Chakraborty et al.

Theorem (Chakraborty et al.)

Quantum spatial search is optimal (i.e.vertex can be found in $\mathcal{O}(\sqrt{n})$) for finding almost all vertices on almost all Erdős-Rényi graphs.

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- Is it possible to "hide" a vertex in a graph?

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Questions:

- Is it possible to "hide" a vertex in a graph?
- Can we obtain common time measurement for all vertices?

Continuous-time quantum spatial search

graph matrix

at $t = 0$ equal superposition of canonic states

$$|\dot{\psi}\rangle = \mathrm{i}(\gamma M + |\omega\rangle\langle\omega|)|\psi\rangle$$

normalizing constant

dyad of search element

Properties of continuous-time quantum spatial search

- Time needed for quantum spatial search is $\mathcal{O}(\frac{1}{|\langle\omega|\lambda_1\rangle|})$.

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- Quantum spatial search is *optimal* if there exists a starting state $|s\rangle$ and the value of γ such that after time $t = \mathcal{O}(\sqrt{n})$ we have

$$P_\omega(t) > \mathrm{const} > 0.$$

Graph spectra

Adjacency matrix A

Laplacian L

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Adjacency matrix A

$\langle v|A|w\rangle = 1$ iff $(v, w) \in E$

Laplacian L

$L = D - A$, where $\langle v|D|v\rangle = \deg(v)$

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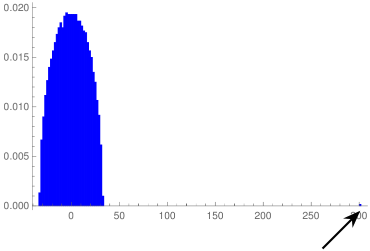

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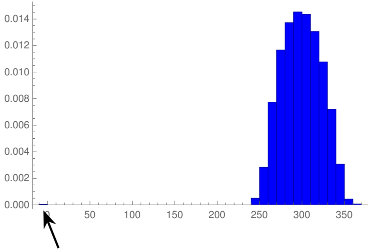

outlying eigenvalue, exactly 0

Adjacency matrix

$$\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
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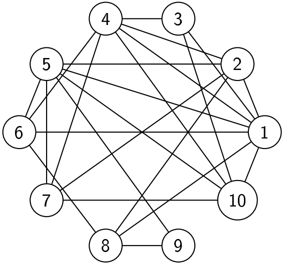

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then

- after time $\mathcal{O}(\sqrt{n})$ we have $P_\omega(t) \geq \frac{1-c}{1+c} - o(1)$.

How did they do it?

Lemma (Chakraborty et al.)

Let

- A *- an adjacency matrix of Erdős-Rényi random graph* $\mathcal{G}(n,p)$,
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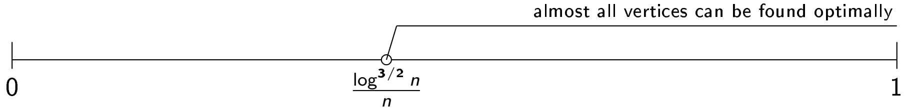

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What do we need?

We want

$$\||\lambda_1\rangle - \frac{1}{\sqrt{n}}\sum_v |v\rangle\|_\infty = o(\frac{1}{\sqrt{n}}).$$

Theorem

Suppose A is an adjacency matrix of a $\mathcal{G}(n,p)$ graph with $p \gg \log^3(n)/n$. Let $|\lambda_1\rangle$ denote the eigenvector corresponding to the largest eigenvalue of A and let $|s\rangle = \frac{1}{\sqrt{n}}\sum_v |v\rangle$. Then

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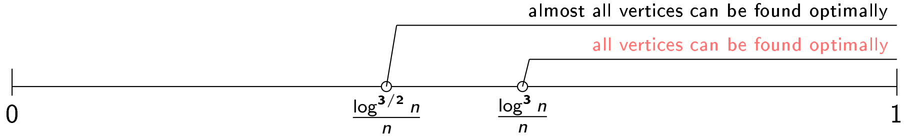
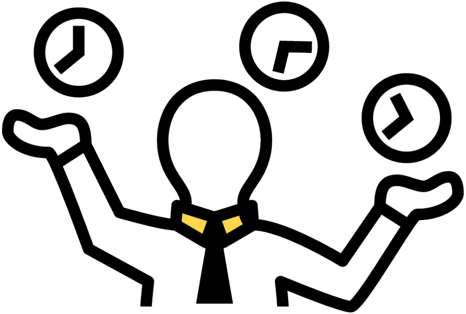


Common time measurement



http://ogaps.tamu.edu/Blog/Blog/September-2017/What-is-Love%E2%80%A6-of-Time

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$\delta = \mathcal{O}(\frac{1}{\sqrt{n}}) \quad \Longrightarrow \quad$ optimal time t may depend on graph.

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Warning

Optimal time may not be the same for all vertices!

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What do we want?

We want $\delta \ll \frac{1}{\sqrt{n}}$ almost surely.

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For $p \gg \log^8(n)/n$

$$\left|\lambda_1\left(\frac{1}{np}A\right) - 1\right| \leq \delta = \mathcal{O}\left(\frac{1}{n\sqrt{p}}\right) = \mathcal{O}\left(\frac{1}{\sqrt{n}\log^4(n)}\right).$$

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All vertices can be found optimally with common measurement time $t=\frac{\pi}{2}\sqrt{n}$ for $p\gg\log^8(n)/n$.

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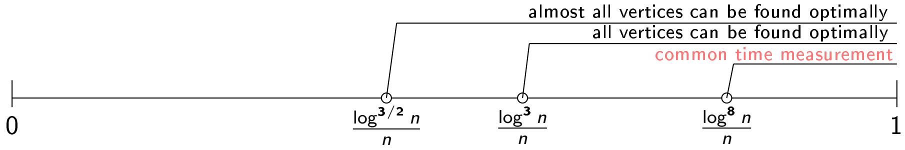

Laplacian matrix

http://niezalezna.pl/205751-bedzie-kolejny-quotmatrixquot

Laplacian matrix

$$\begin{bmatrix} 7 & -1 & -1 & -1 & -1 & -1 & 0 & -1 & 0 & -1 \\ -1 & 5 & 0 & -1 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 0 & 3 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 6 & 0 & -1 & -1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 6 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & -1 & 4 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 & 4 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & -1 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 2 & 0 \\ -1 & 0 & -1 & -1 & -1 & 0 & -1 & 0 & 0 & 5 \end{bmatrix}$$

Laplacian matrix

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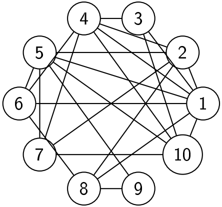

Laplacian matrix

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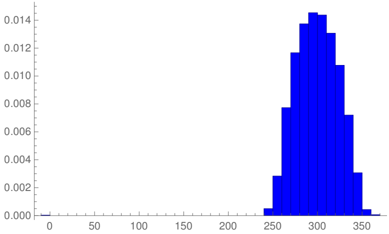

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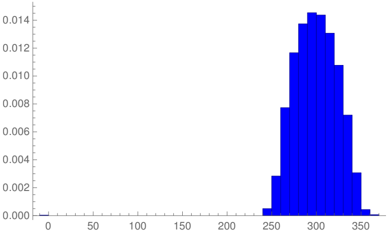

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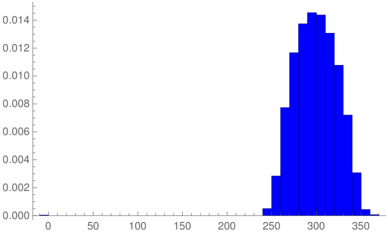

- outlying eigenvalue equals 0,
- spectrum of $\frac{L}{np}$ converge to 1

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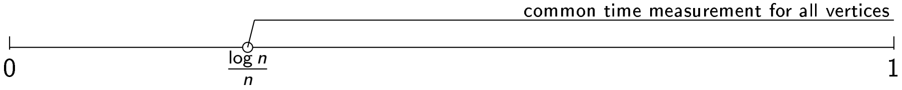

How about hiding...

veritces?

https://cats.lovetoknow.com/cat-pictures-slideshows/cats-who-fail-at-hiding

When can we hide vertices?

Remember that...

For $p < \frac{(1-\varepsilon)\log(n)}{n}$ a graph almost surely contains isolated vertices.

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- Laplacian matrix

there exist vertices which cannot be found in $o(n)$.

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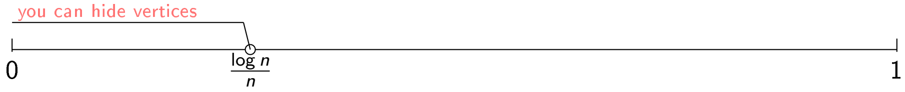
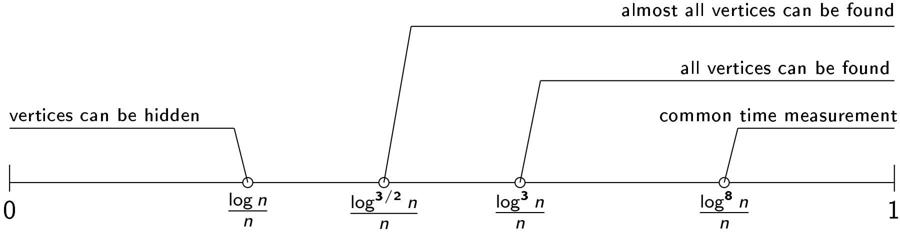


Summary

https://academichelp.net/academic-assignments/essay/write-summary-essay.html

- Adjacency matrix



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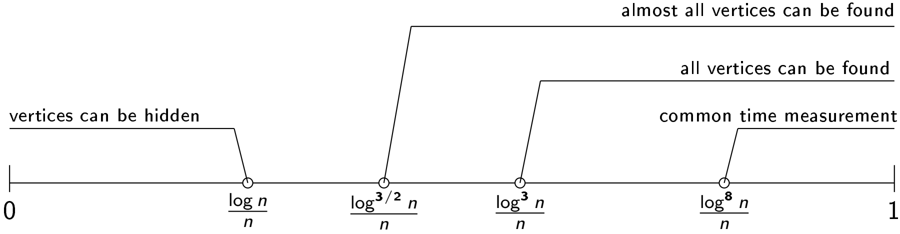

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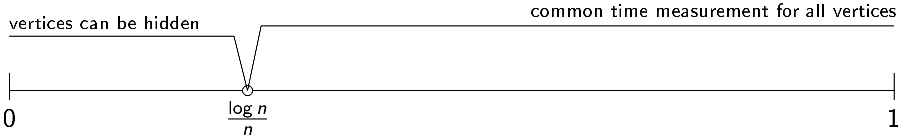

https://depositphotos.com/62068585/stock-photo-man-with-a-road-sign.html

What can be done?

What about p?

- Try to say something about $\frac{(1+\varepsilon)\log n}{n} < p \ll (\log^8(n)/n)$ for adjacency matrix.

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- Try to say something about $\frac{(1+\varepsilon)\log n}{n} < p \ll (\log^8(n)/n)$ for adjacency matrix.

Other?

- Different random graph models,
- Various kinds of noise.

Thank you for your attention!

Any questions?

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