shorter proofs for G-R-O-U-N-D-S-T-A-T-E-C-O-N-N-E-C-T-I-V-I-T-Y

shorter
proofs

for

G-
R-O-
U-N-D-
S-T-A-T-
E-C-O-N-N-
E-C-T-I-V-I-T-Y

libor caha
daniel nagaj
martin schwarz

IoP SAS
Smolenice
CEQIP 2018

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CrossMark

Shorter unentangled proofs for ground state connectivity

Libor Caha[1] · **Daniel Nagaj**[1] · **Martin Schwarz**[2]

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Abstract Can one considerably shorten a proof for a quantum problem by using a protocol with a constant number of unentangled provers? We consider a frustration-free variant of the QCMA-complete ground state connectivity (GSCON) problem for a system of size n with a proof of superlinear size. We show that we can shorten this proof in QMA(2): There exists a two-copy, unentangled proof with length of order n, up to logarithmic factors, while the completeness–soundness gap of the new protocol becomes a small inverse polynomial in n.

Keywords Quantum complexity · QMA(2) · Unentanglement · Short proofs · Ground state connectivity problem (GSCON)

1 Introduction: unentangled provers and short proofs

While entanglement is essential for quantum algorithms, *unentanglement* can also be an interesting resource. In quantum complexity, such a guarantee about a purported proof can significantly improve the power of a verifier. Blier and Tapp [3] discovered that two unentangled copies of a *short* witness of the type

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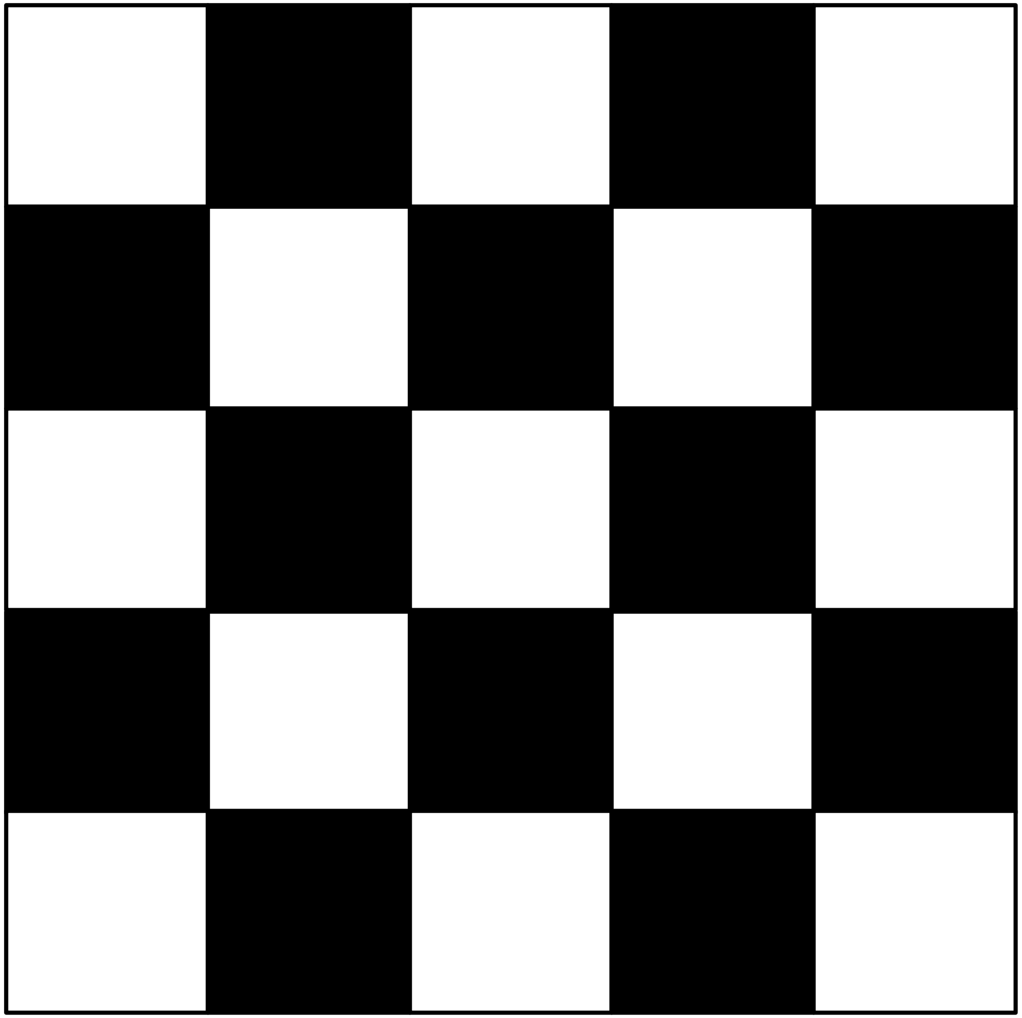
- Puzzles and proofs

- 5 queens, > 10 solutions?

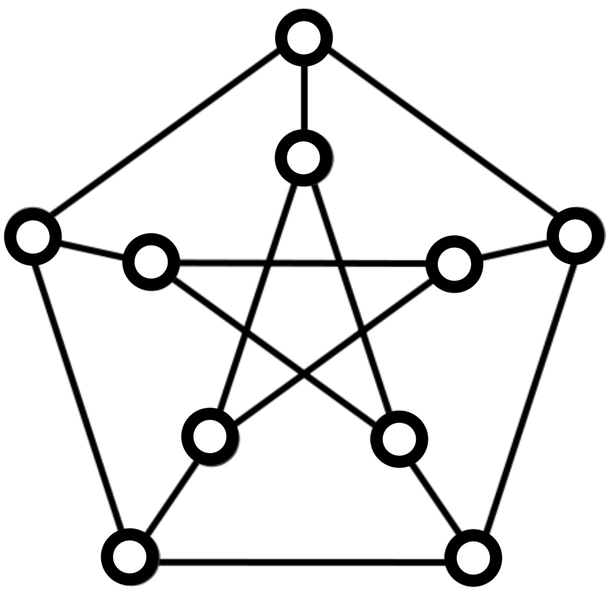


- NP: checkable proofs

traveling salesman: a list of cities

sudoku: fill the boxes with numbers

graph 3-coloring: vertex colors



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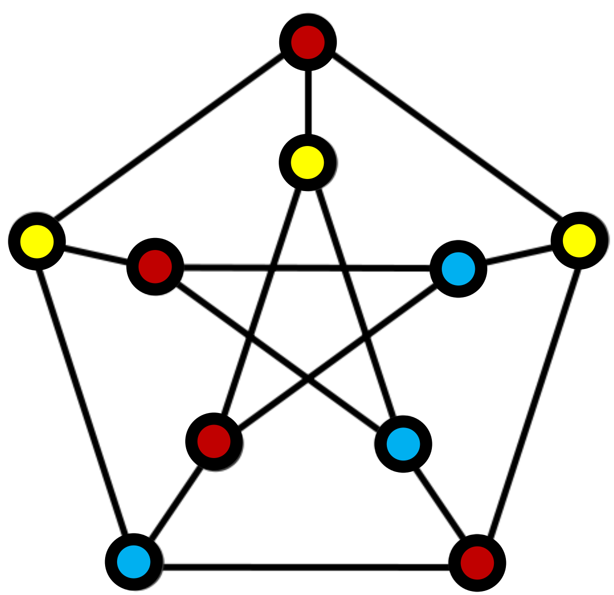


What to do with a proof?

convince me!

NP

Verify.

say what?

What if we only have probabilistic checks?

completeness (YES)

convince me

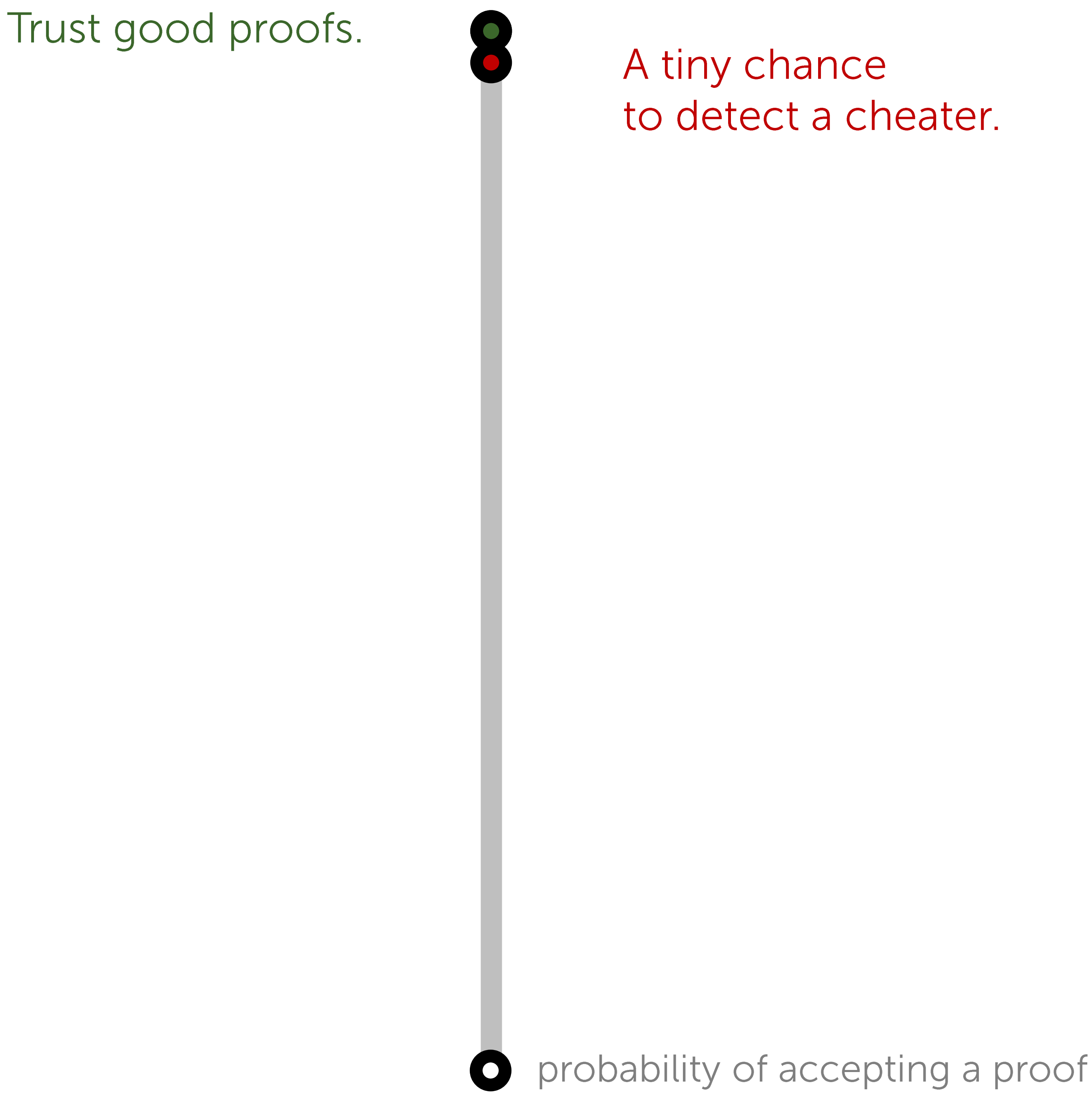
stay skeptical

soundness (NO)

Carefully verify!

MA

probability of accepting a proof


Trust good proofs.
A tiny chance
to detect a cheater.
probability of accepting a proof

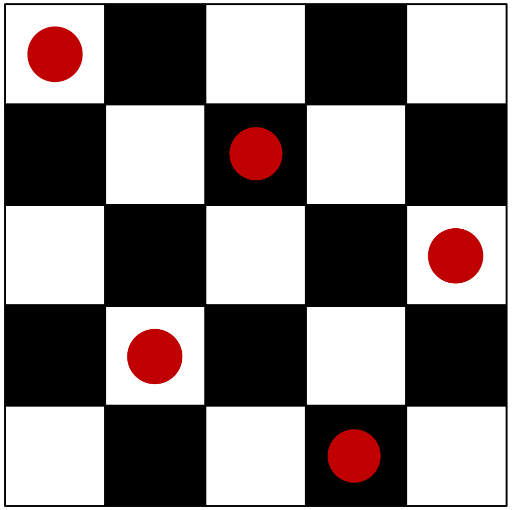
Proofs and checks

deterministic / randomized

full / partial

classical / quantum

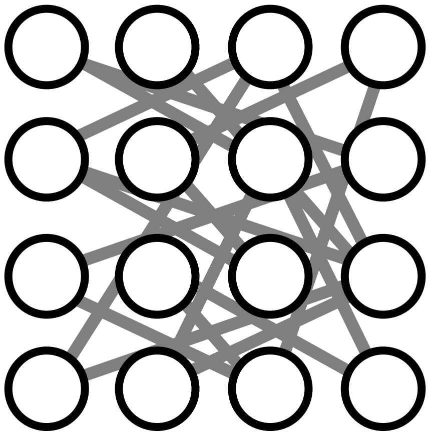
general / with promises



+ symmetries = 10.

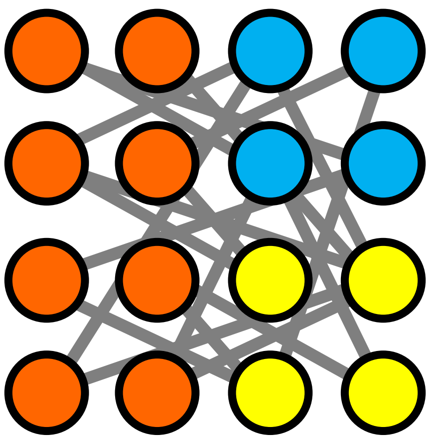
Quantum proofs for G3C

a “long” classical
proof: the colors



Quantum proofs for G3C

a "long" classical proof: the colors



Quantum proofs for G3C

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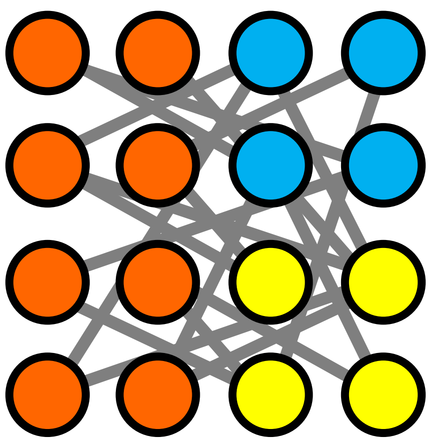

a “short” quantum proof: superpose

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}|i\rangle|c_i\rangle$$

Quantum proofs for G3C

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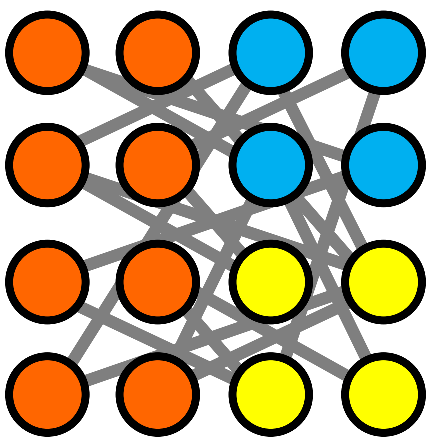

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How short is it?

Quantum proofs for G3C

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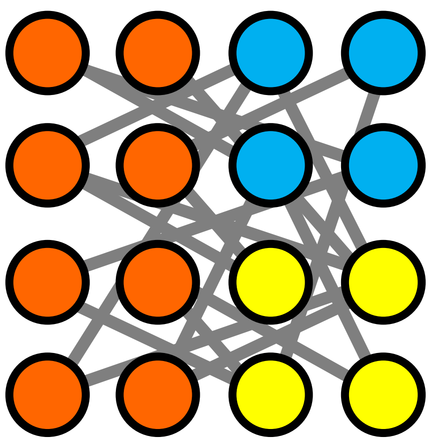


a “short” quantum proof: superpose

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} |i\rangle |c_i\rangle$$

Can you verify it?

Promise: unentanglement?

QMA(2): the power of unentanglement

2 short
unentangled proofs

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}|i\rangle|c_i\rangle$$

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}|i\rangle|c_i\rangle$$

QMA(2): the power of unentanglement

checking 2 short unentangled proofs

feasibility

compatibility

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}|i\rangle|c_i\rangle$$

$$\updownarrow$$

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}|i\rangle|c_i\rangle$$

QMA(2): the power of unentanglement

checking 2 short
unentangled proofs

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} |i\rangle |c_i\rangle$$

feasibility
compatibility
Fourier

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} |i\rangle |c_i\rangle$$

QMA(2): the power of unentanglement

[Blier & Tapp]
[Beigi & Shor]
[Aaronson et al.]
[Le Gall et al.]

checking 2 short unentangled proofs

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} |i\rangle |c_i\rangle$$

feasibility
compatibility
Fourier

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} |i\rangle |c_i\rangle$$

state of the art
2 log-size proofs, constant $c-s$ gap

Quantum problems & (quantum) proofs

is the ground energy < E (or > *F*)?

Quantum problems & (quantum) proofs

is the ground energy < E (or > *F*)?

a proof?
the g. s.

verification?
measurement

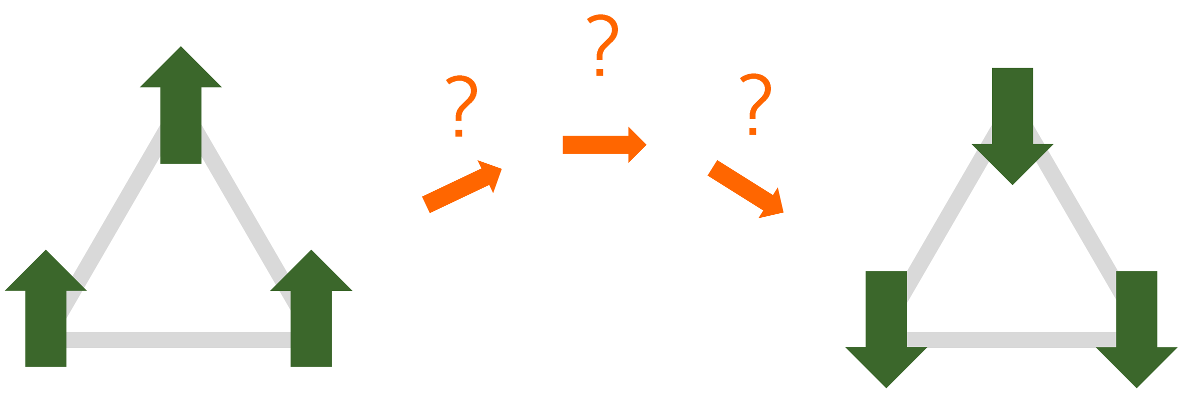
Shorter proofs for a quantum problem

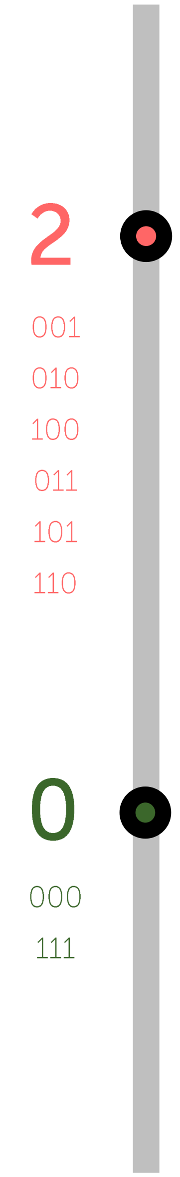
encode some quantum proof
in a verifiable superposition?

$$\frac{1}{\sqrt{2m}} \sum_{i=1}^{2m} |i\rangle |\psi_i\rangle$$

A low energy traversal

ferro Hamiltonian $\frac{1}{2}\sum_{i\neq j}(\mathbb{I} - Z_i Z_j)$





1 & 2-local gates, keep E < 1.5

GSCON traversing a ground space

[Sikora & Gharibian]

H, in, out, bounds

a sequence of 1 & 2-local gates?

GSCON traversing a ground space

[Sikora & Gharibian]
[Gosset, Mehta, Vidick]

H, in, out, bounds

a sequence of 1 & 2-local gates?

classical description

state of the art

QCMA complete

commuting Hamiltonians

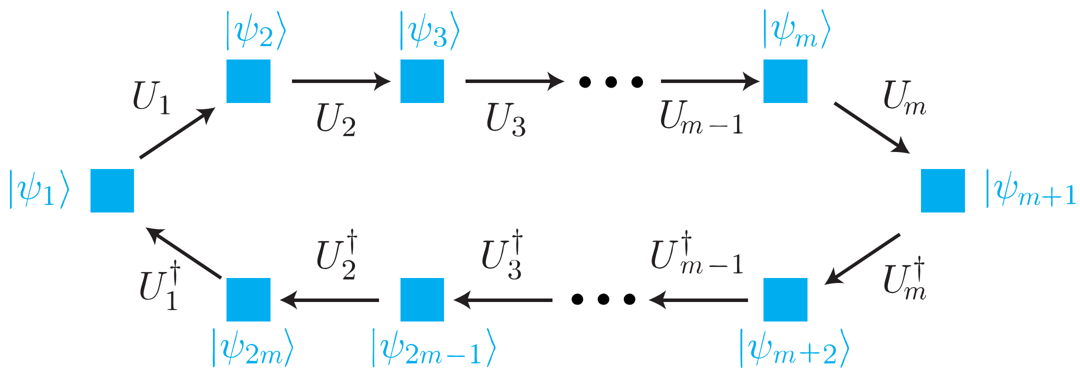
Shorter proofs for a quantum problem

encode a quantum proof
(a long sequence of states)
in a verifiable superposition?

$$\frac{1}{\sqrt{2m}}\sum_{i=1}^{2m}|i\rangle|\psi_i\rangle$$

A GSCON traversal path as a superposition

a cycle

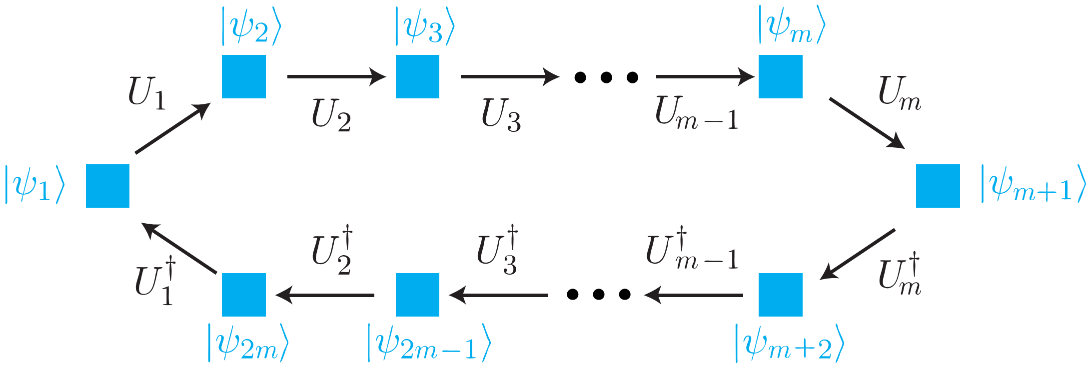


of states

$$\frac{1}{\sqrt{2m}} \sum_{i=1}^{2m} |i\rangle |\psi_i\rangle$$

A GSCON traversal path encoded in superpositions

a cycle

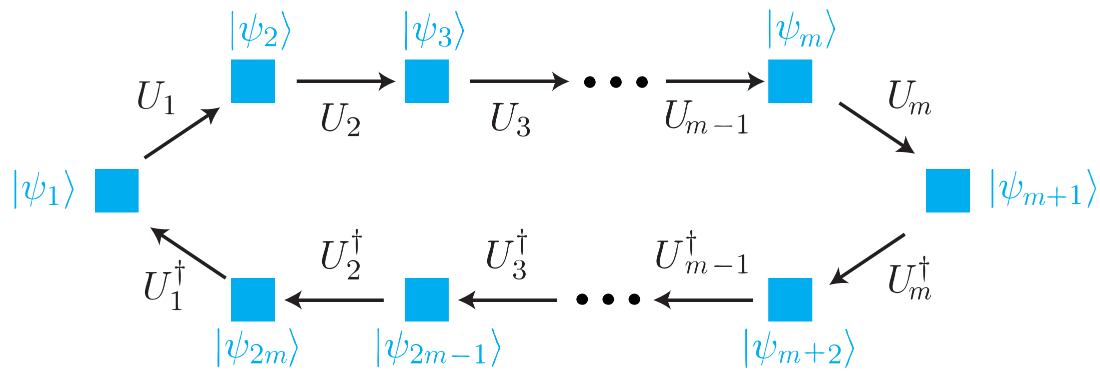


of states $\frac{1}{\sqrt{2m}} \sum_{i=1}^{2m} |i\rangle |\psi_i\rangle$

of gates $\frac{1}{\sqrt{2m}} \sum_{i=1}^{2m} |i\rangle |u_i\rangle$

A GSCON traversal path encoded in superpositions

a cycle



of states $\frac{1}{\sqrt{2m}} \sum_{i=1}^{2m} |i\rangle|\psi_i\rangle$

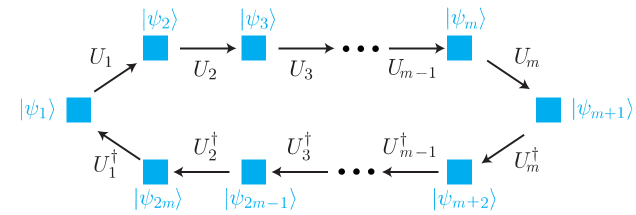
$$\sum_{i=1}^{2m} |i+1\rangle\langle i| \otimes U_i$$

shift invariant

of gates $\frac{1}{\sqrt{2m}} \sum_{i=1}^{2m} |i\rangle|u_i\rangle$

The testing of the path

2 copies of the cycle states & gates



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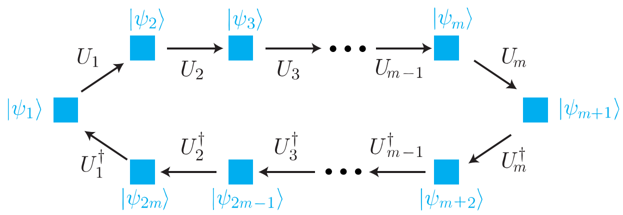
consistency of U's

The testing of the path

2 copies of the cycle states & gates



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consistency of *U*'s, all of the *U*'s

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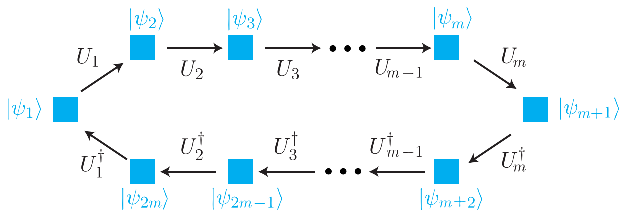

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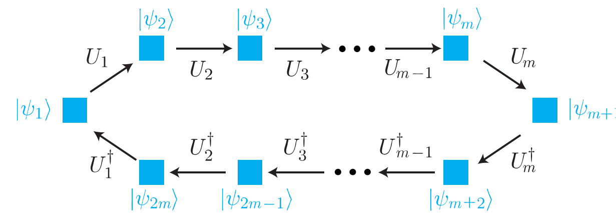
consistency of *U*'s, all of the *U*'s

consistency of states

The testing of the path

2 copies of the cycle states & gates



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shift invariant

$$\sum_{i=1}^{2m}|i+1\rangle\langle i|\otimes U_i$$

consistency of U's, all of the U's

consistency of states

shift invariance

The testing of the path

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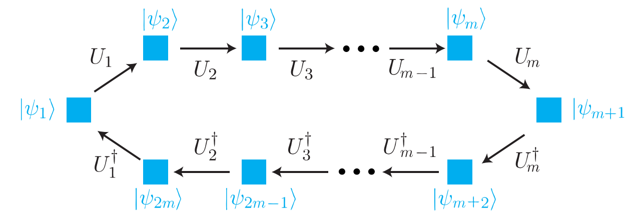

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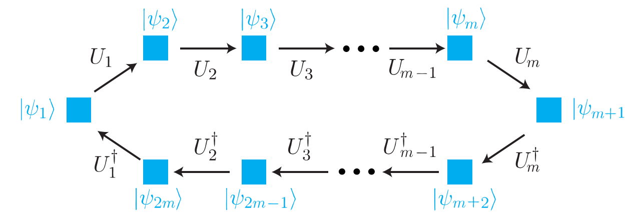

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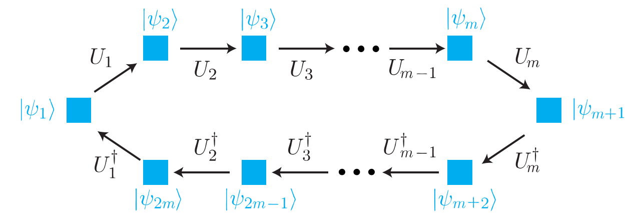

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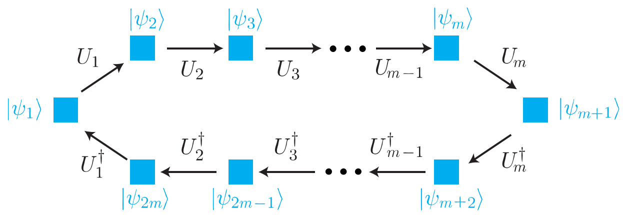

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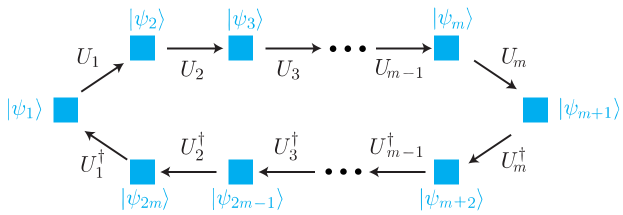

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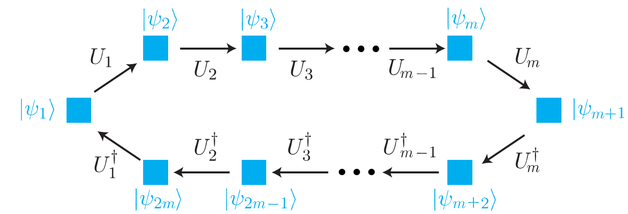

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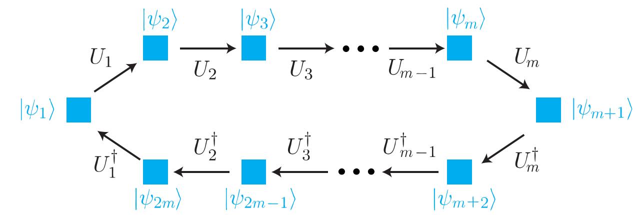

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consistency of U's, all of the U's

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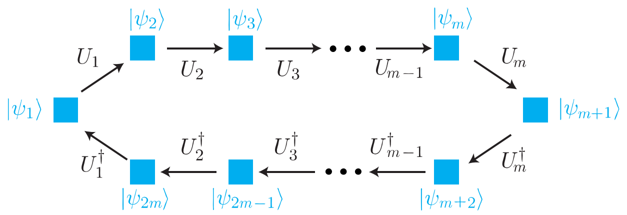
The testing of the path

2 copies of the cycle
states & gates



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consistency of U's, all of the U's

consistency of states

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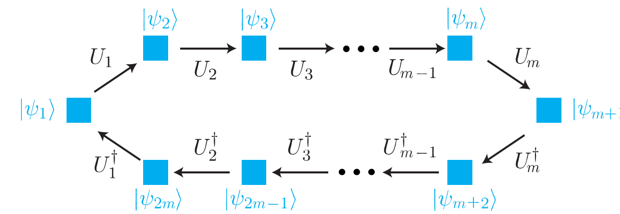
The testing of the path

2 copies of the cycle states & gates



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shift invariant

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consistency of U's, all of the U's

consistency of states

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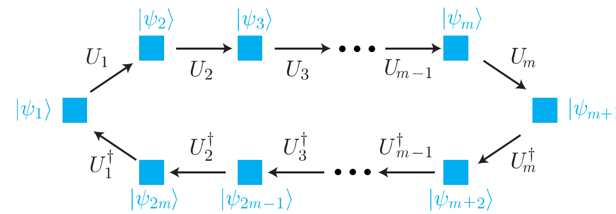
The testing of the path

2 copies of the cycle states & gates



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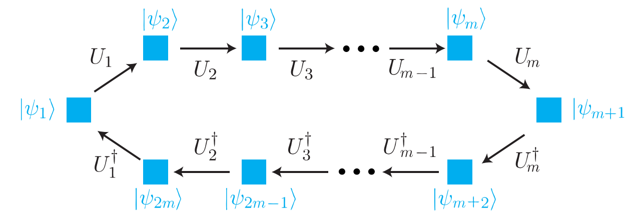
The testing of the path

2 copies of the cycle states & gates



$$\frac{1}{\sqrt{2m}}\sum_{i=1}^{2m}|i\rangle|\psi_i\rangle \qquad \frac{1}{\sqrt{2m}}\sum_{i=1}^{2m}|i\rangle|u_i\rangle$$

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shift invariant

$$\sum_{i=1}^{2m}|i+1\rangle\langle i|\otimes U_i$$

consistency of U's, all of the U's

consistency of states

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The testing of the path

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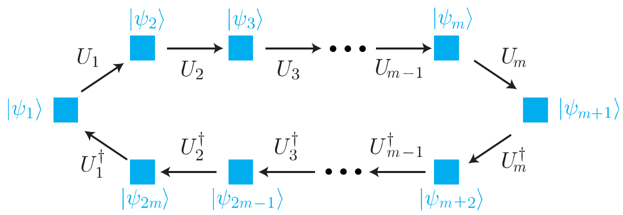

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low energy

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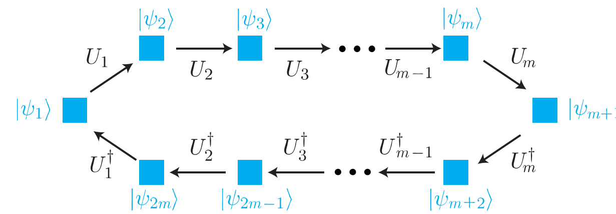

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low energy, start & finish

Reliability
$n \log m$
of the short
unentangled
proofs?

1/poly(n)

A tiny
chance
to detect
a cheater.

probability of accepting a proof

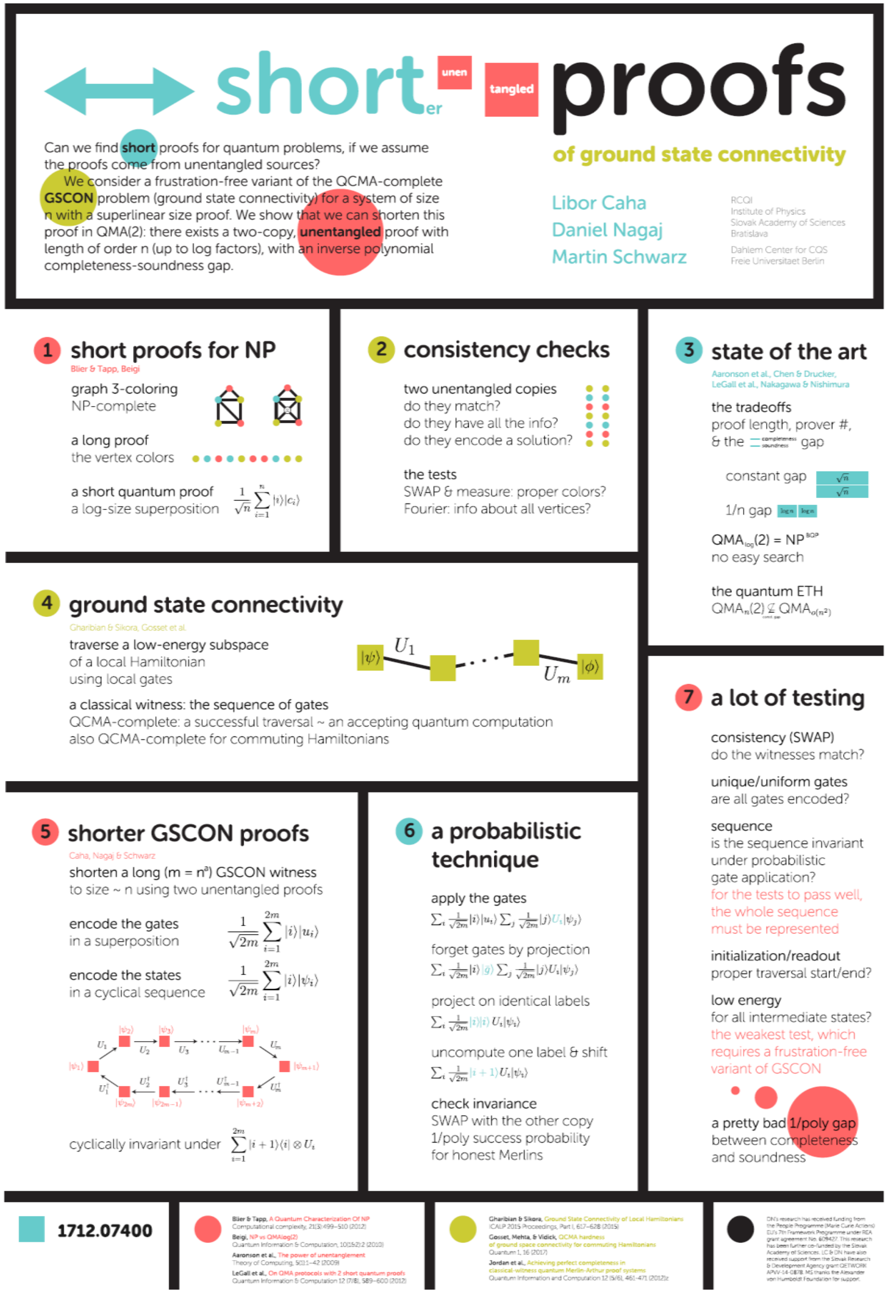
Reliability of the short ($n \log m$) unentangled proofs?

Is it cool?

A tiny ($1/\mathrm{poly}(n)$) chance to detect a cheater.

For looong ($m = n^{a>1}$) sequences.

probability of accepting a proof



A better low energy test?

A better gap?

Counting problems?

Something QMA hard?

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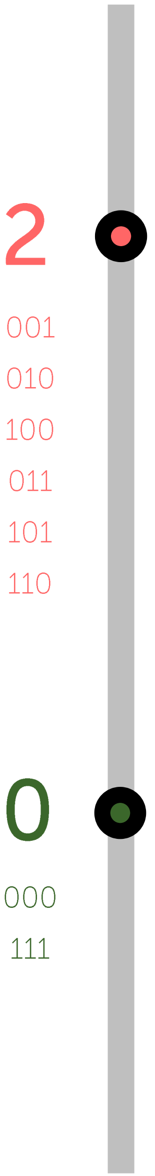
A low energy traversal



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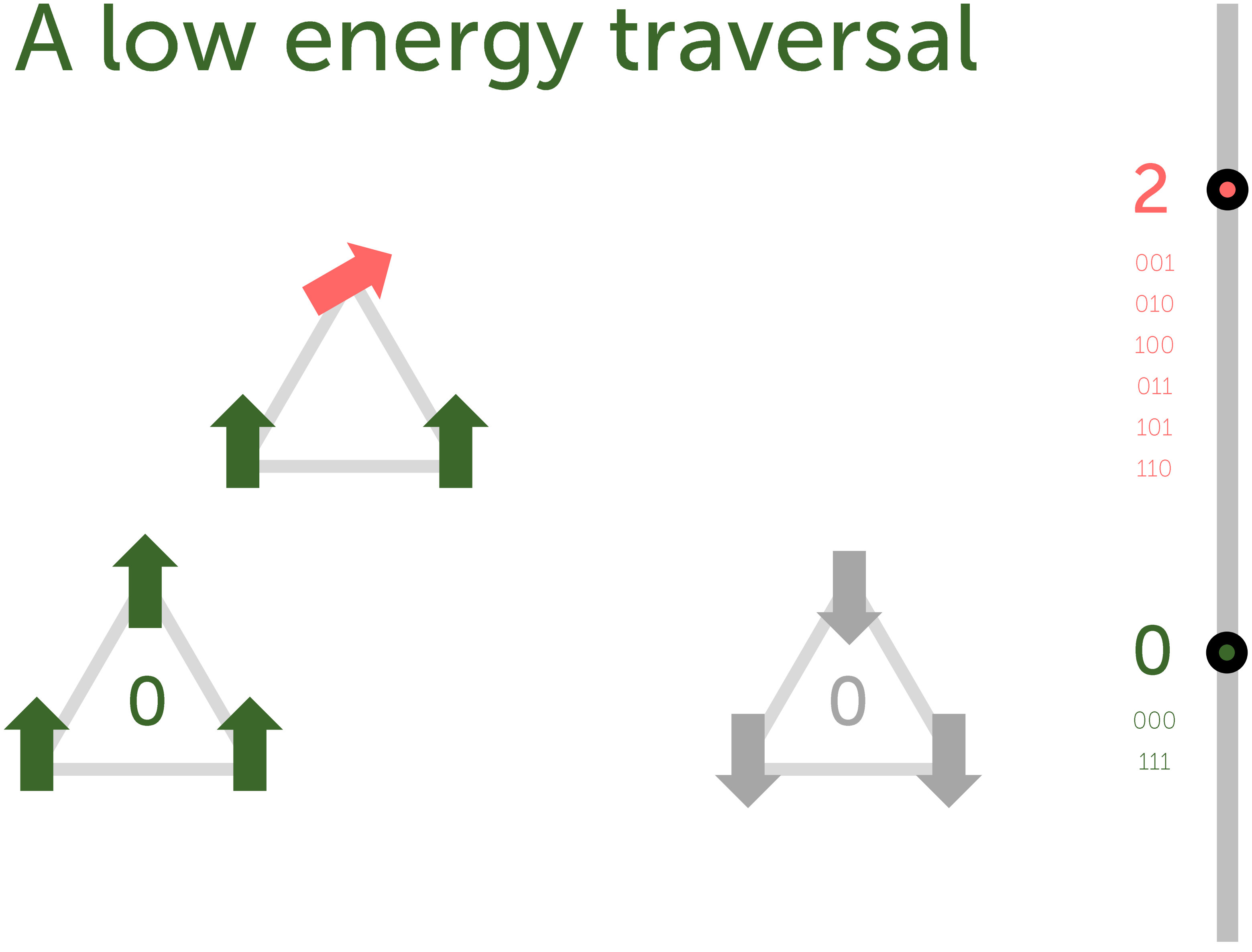

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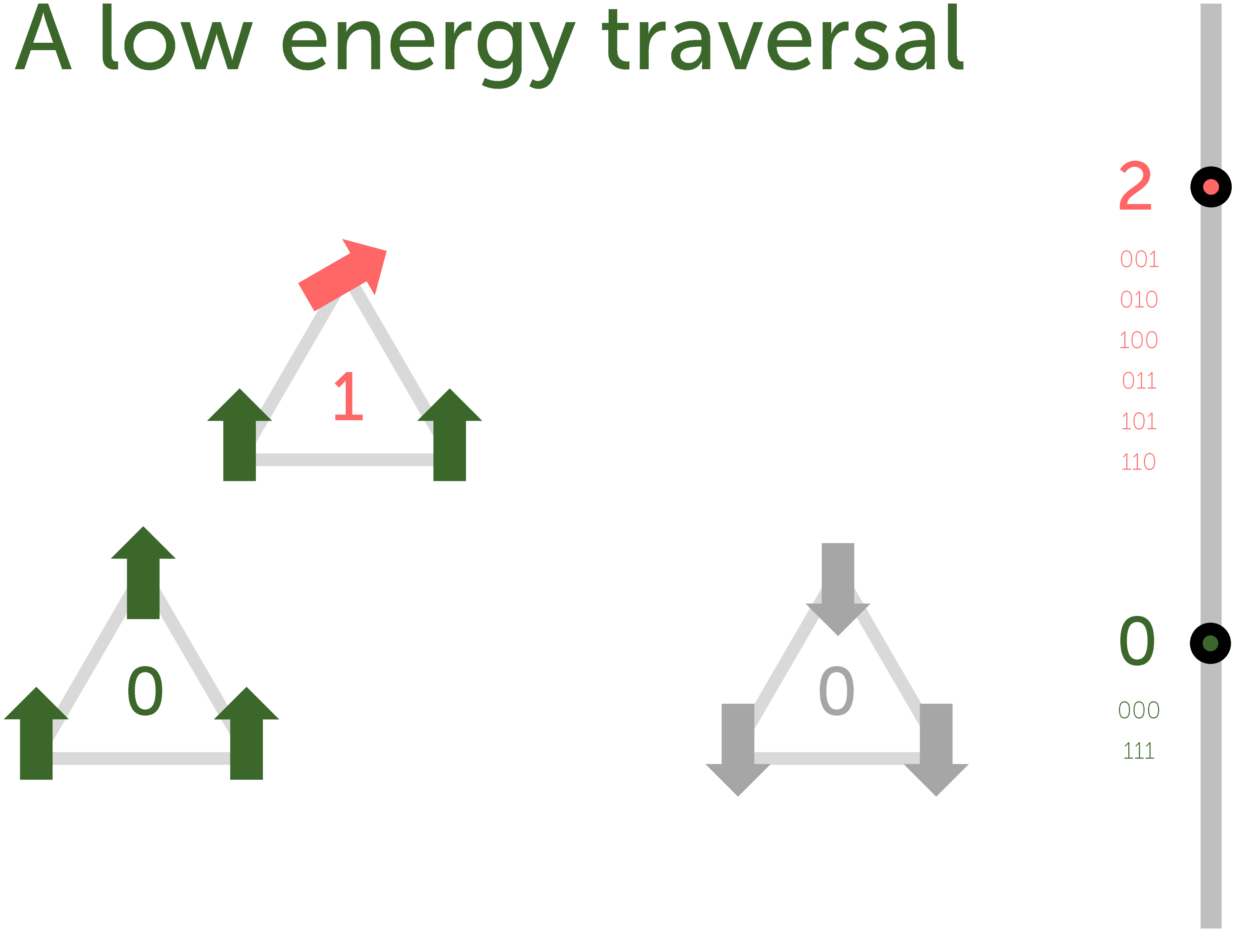

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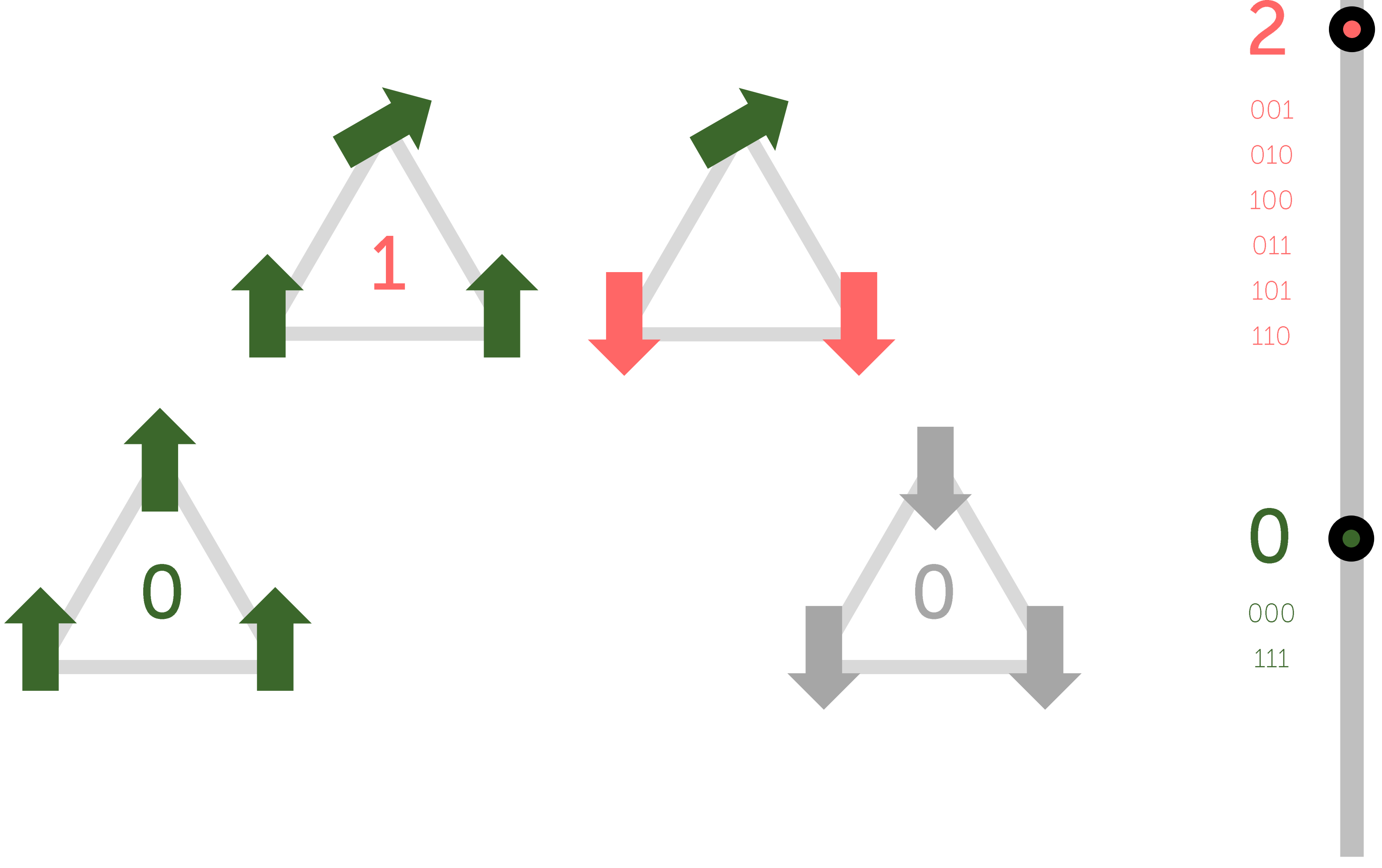

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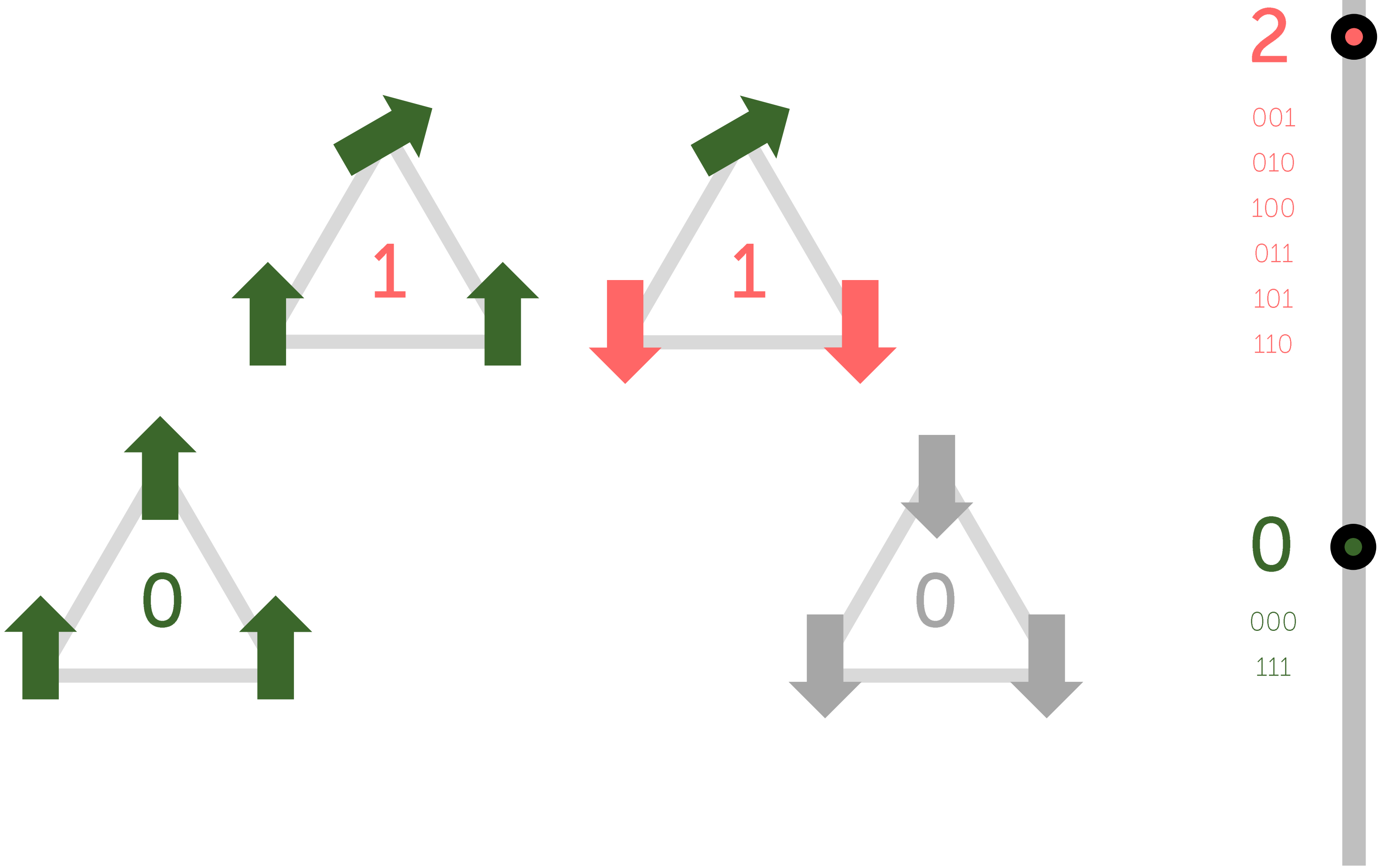

A low energy traversal