



Resetting uncontrolled quantum systems

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MN, arXiv:1710.02470

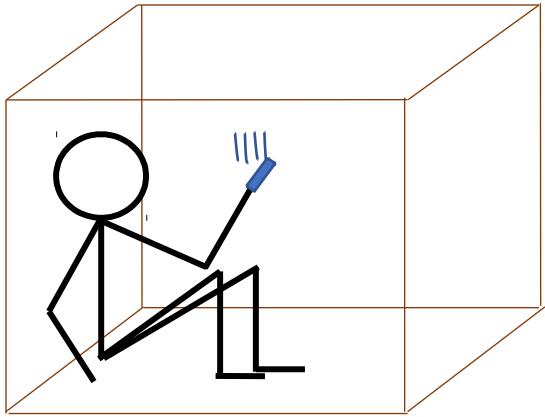


Time-warp

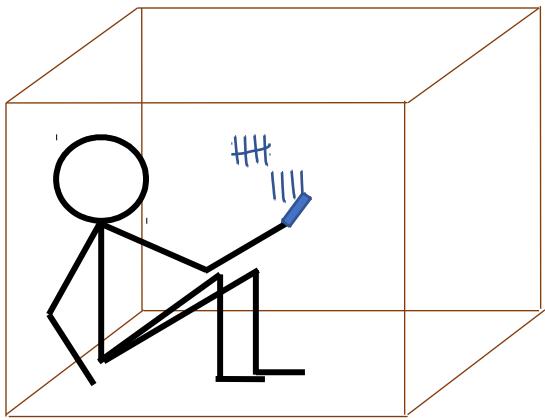
noun | \ 'tīm 'wōrp \

Definition of TIME-WARP

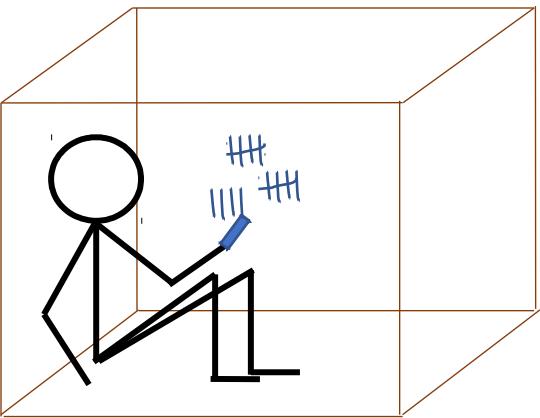
1. an anomaly, discontinuity, or suspension held to occur in the progress of time



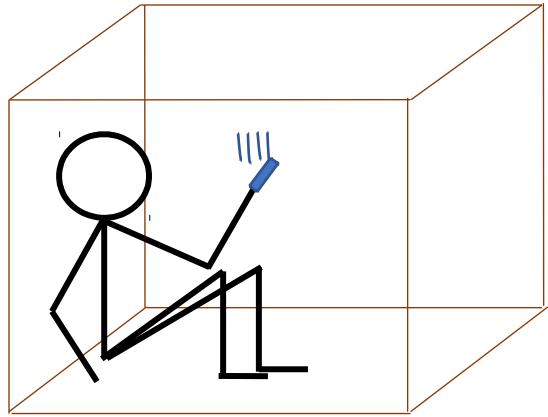
$t = 0$



$t = \frac{1}{\text{hour}}$



$t = 2 \frac{1}{2}$ hours

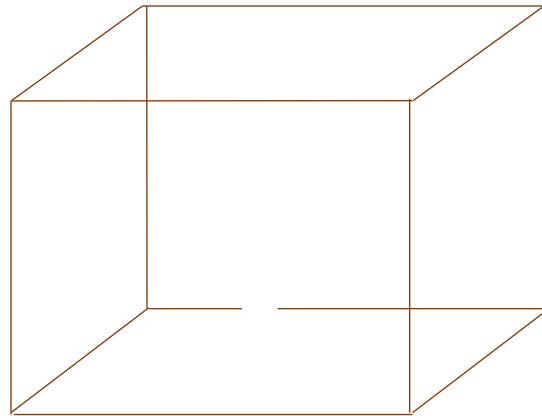


$t = 0$



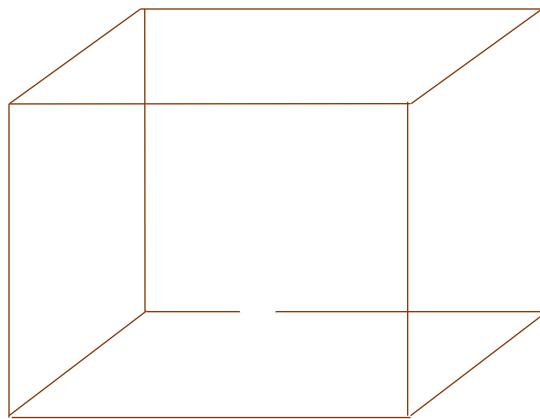
Time warp for two hours

What should we find in the box if we are to claim a time warp experience?



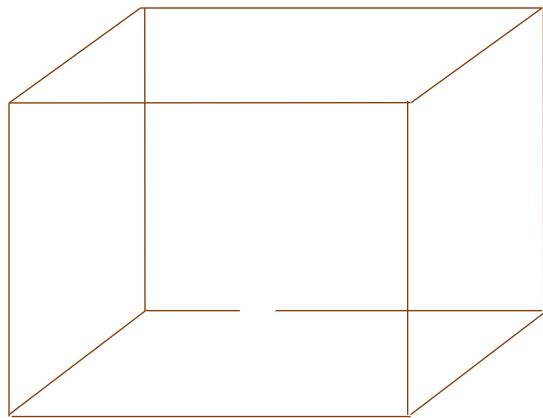
$$t = 2 \text{ hours}$$

Not expected



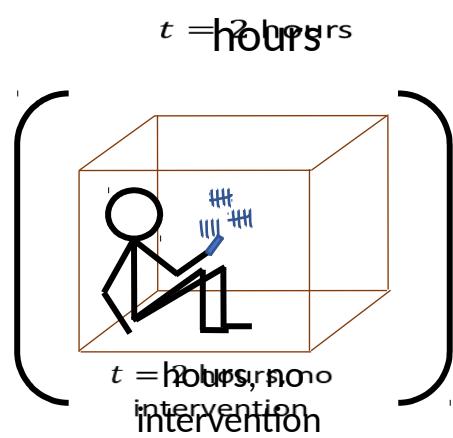
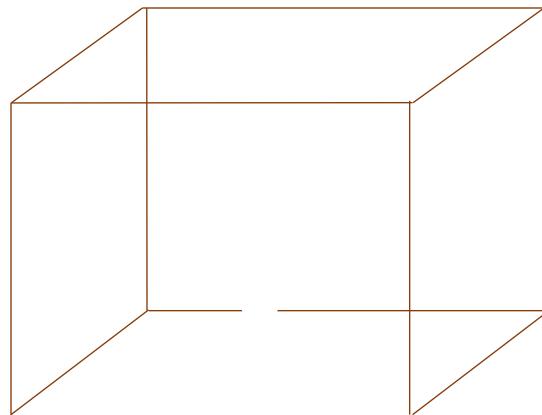
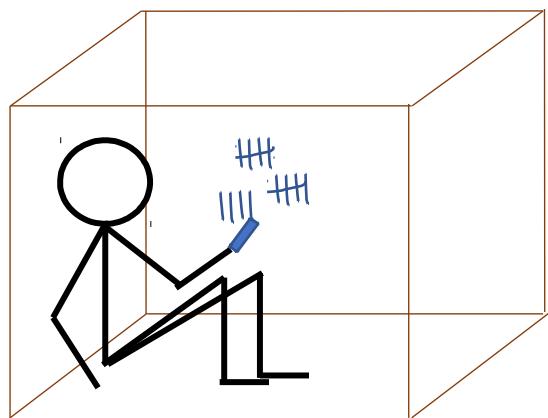
$$t = 2 \text{ hours}$$

Not expected

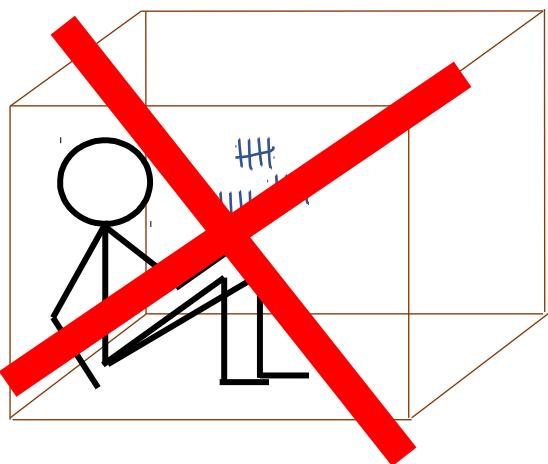


$$t = 2 \text{ hours}$$

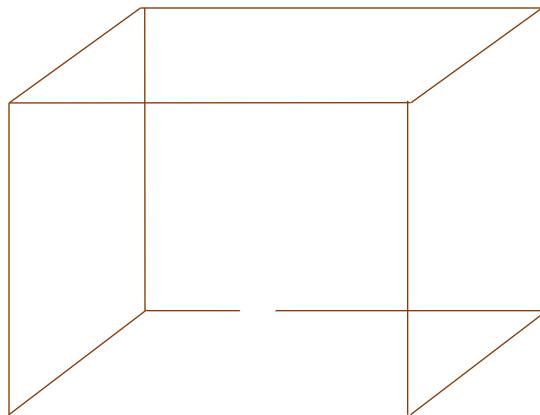
Not expected



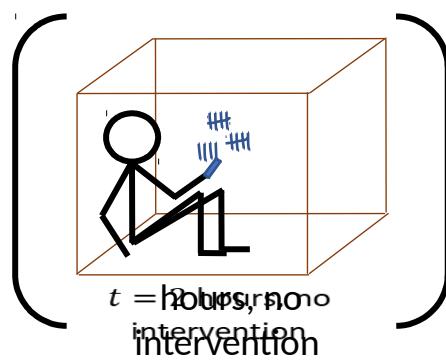
Not expected



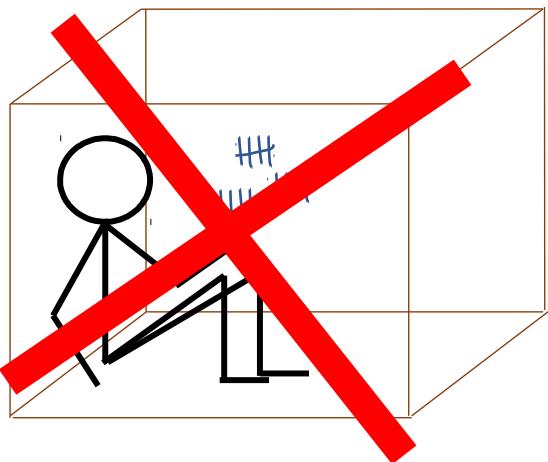
Expected



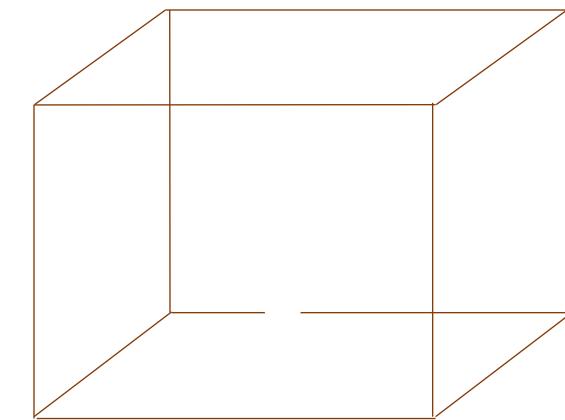
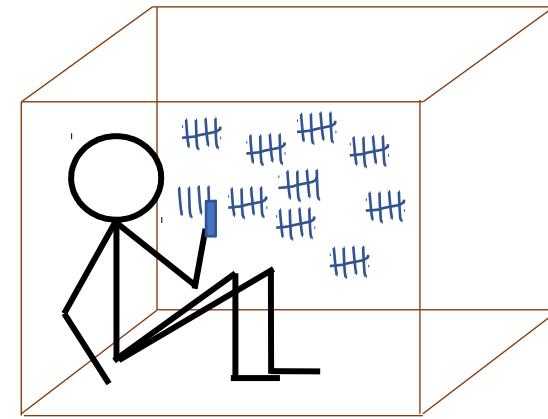
$t = 2 \text{ hours}$



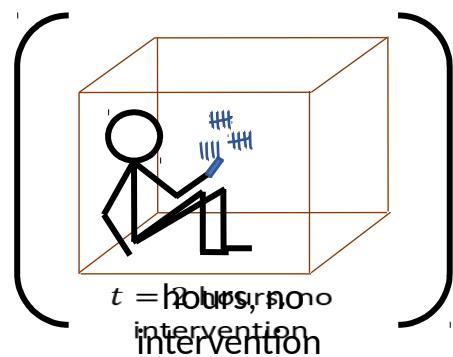
Not expected



Expected

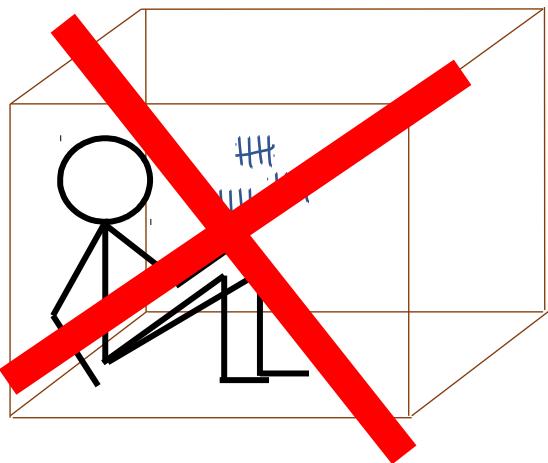


$t = 2 \text{ hours}$

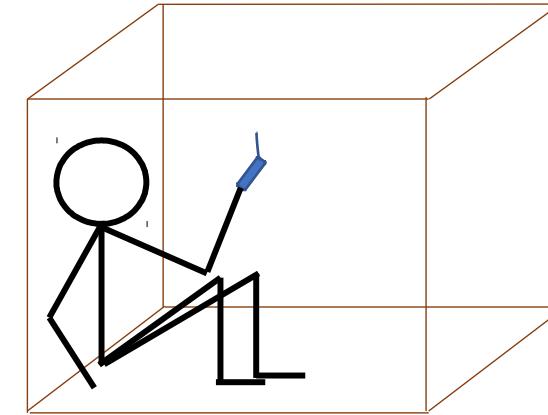
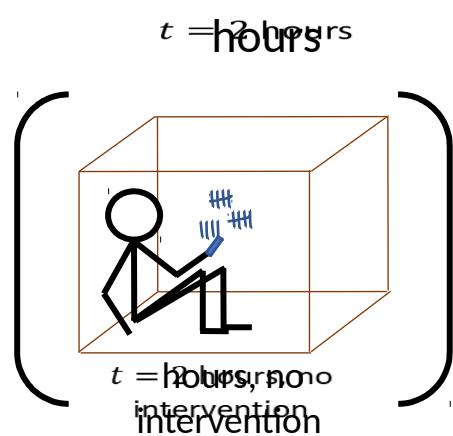
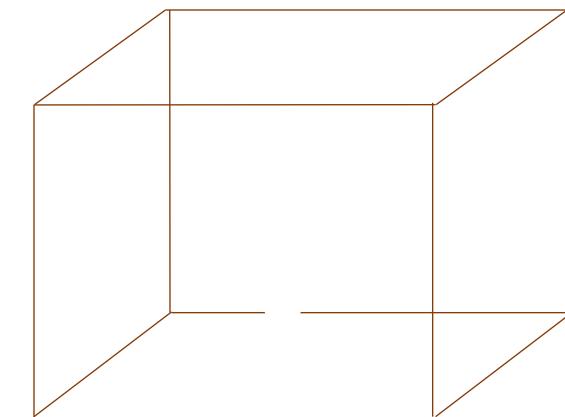
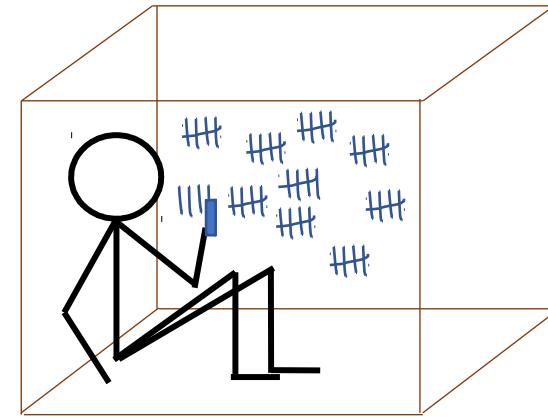


$t = 2 \text{ hours, no}$
intervention

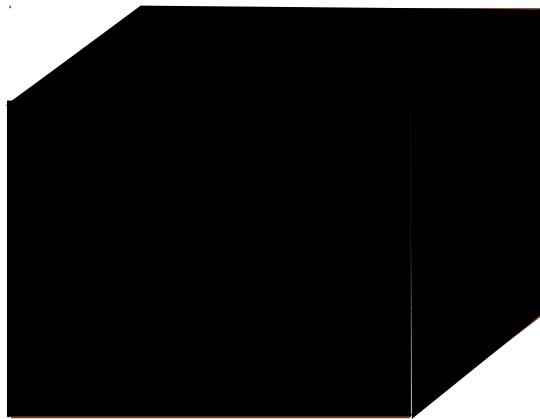
Not expected



Expected

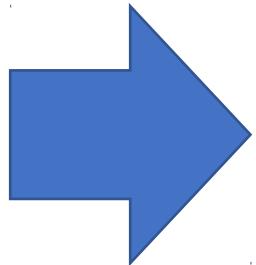
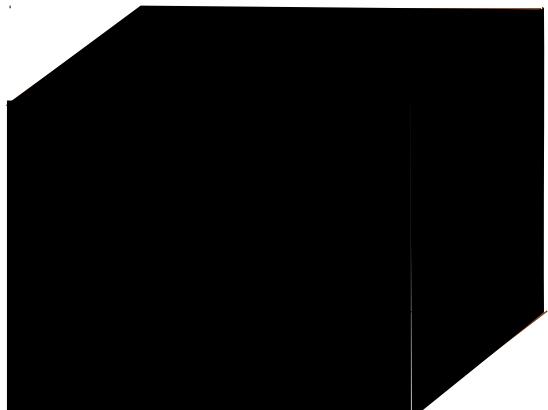


Time warp: operational definition

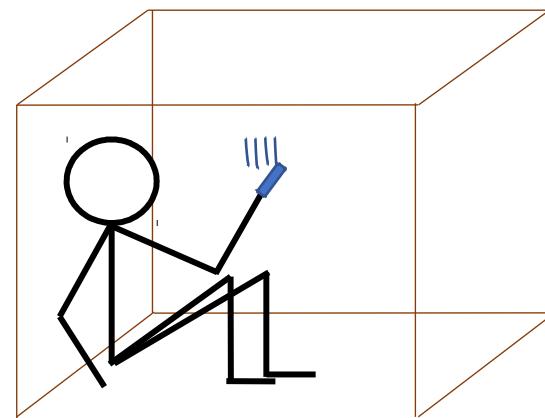


$\{\psi(t): t\}$

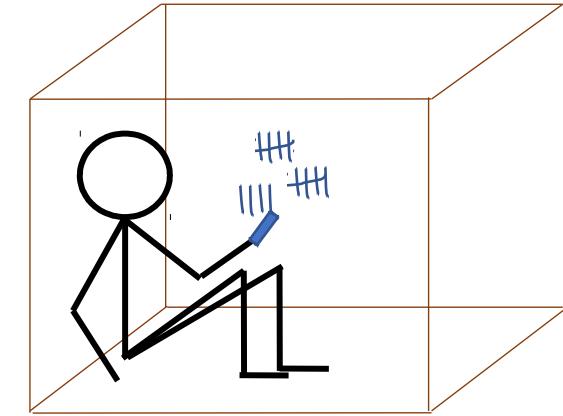
Time warp: operational definition



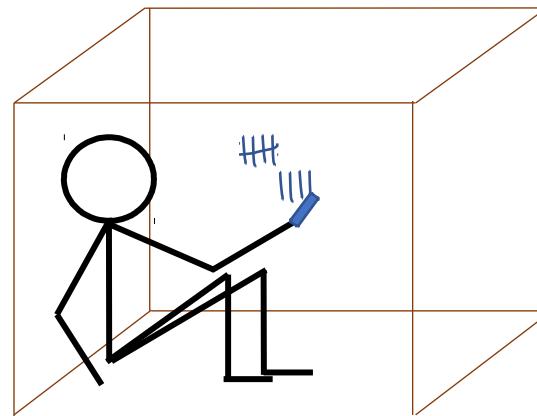
$\{\psi(t): t\}$



$\psi(0)$



$\psi(1)$



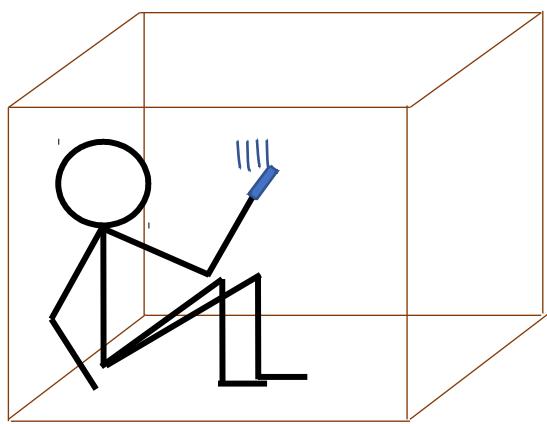
$\psi(2)$

Time warp: operational definition

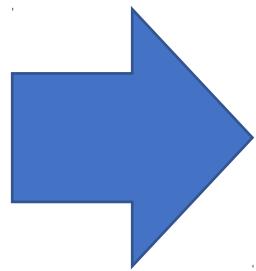


Time warp protocol for $[0, \tau]$

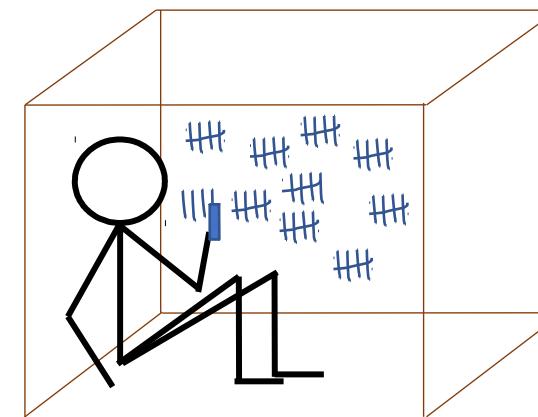
Time warp: operational definition



$\psi(0)$



Time warp
for $t \in [0, \tau]$



$\psi(\tau')$
 $\tau' \neq \tau$

$\{\psi(t): t\}$



A brief history of time warp

King Raivata and
princess Revati
(400BC?)





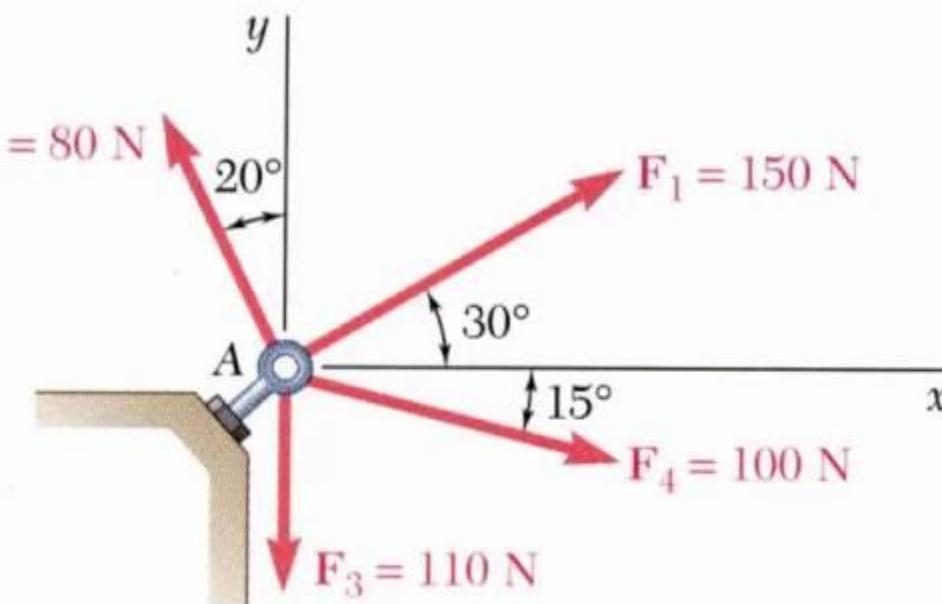
Peter Damian
(1007-1072)

The Time Machine



H. G. Wells

(first edition in 1895)



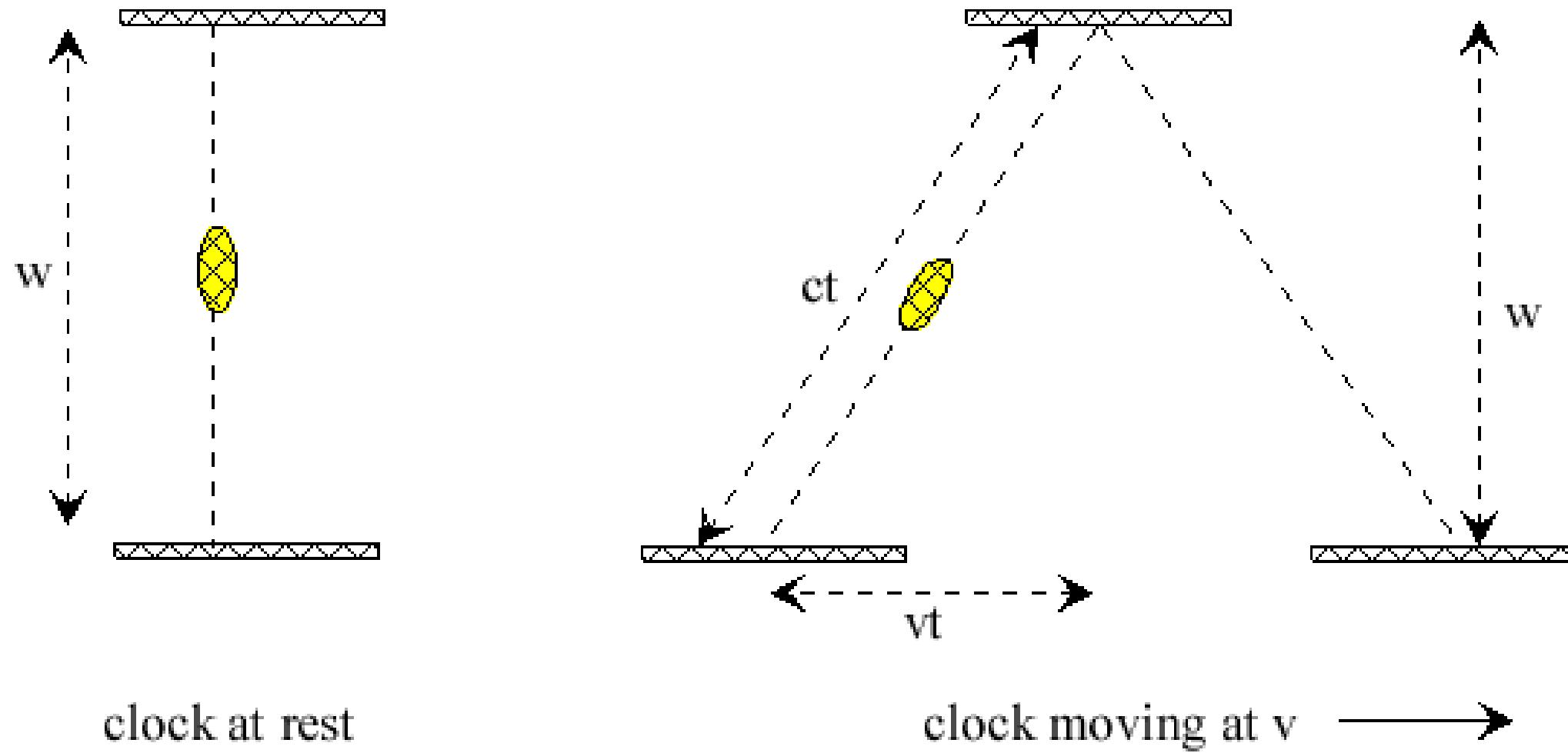
Four forces act on bolt A as shown. Determine the resultant of the force on the bolt.

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

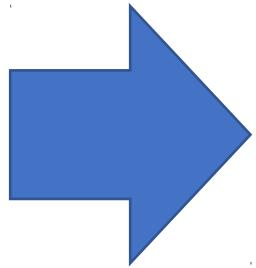
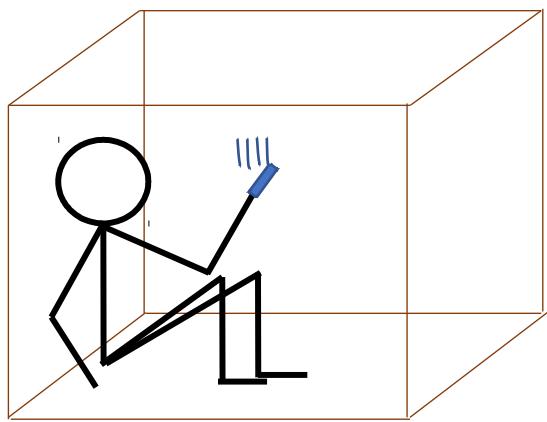


Warping time physically

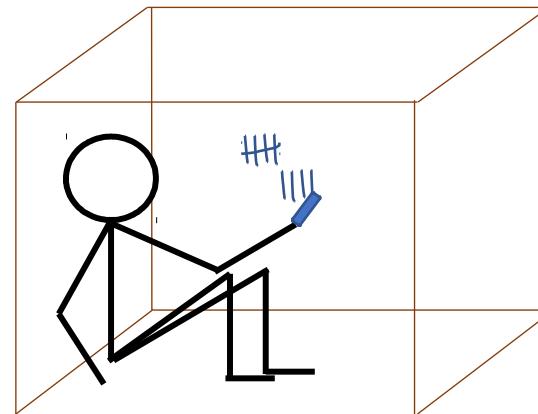
mirror



Time warp in special relativity

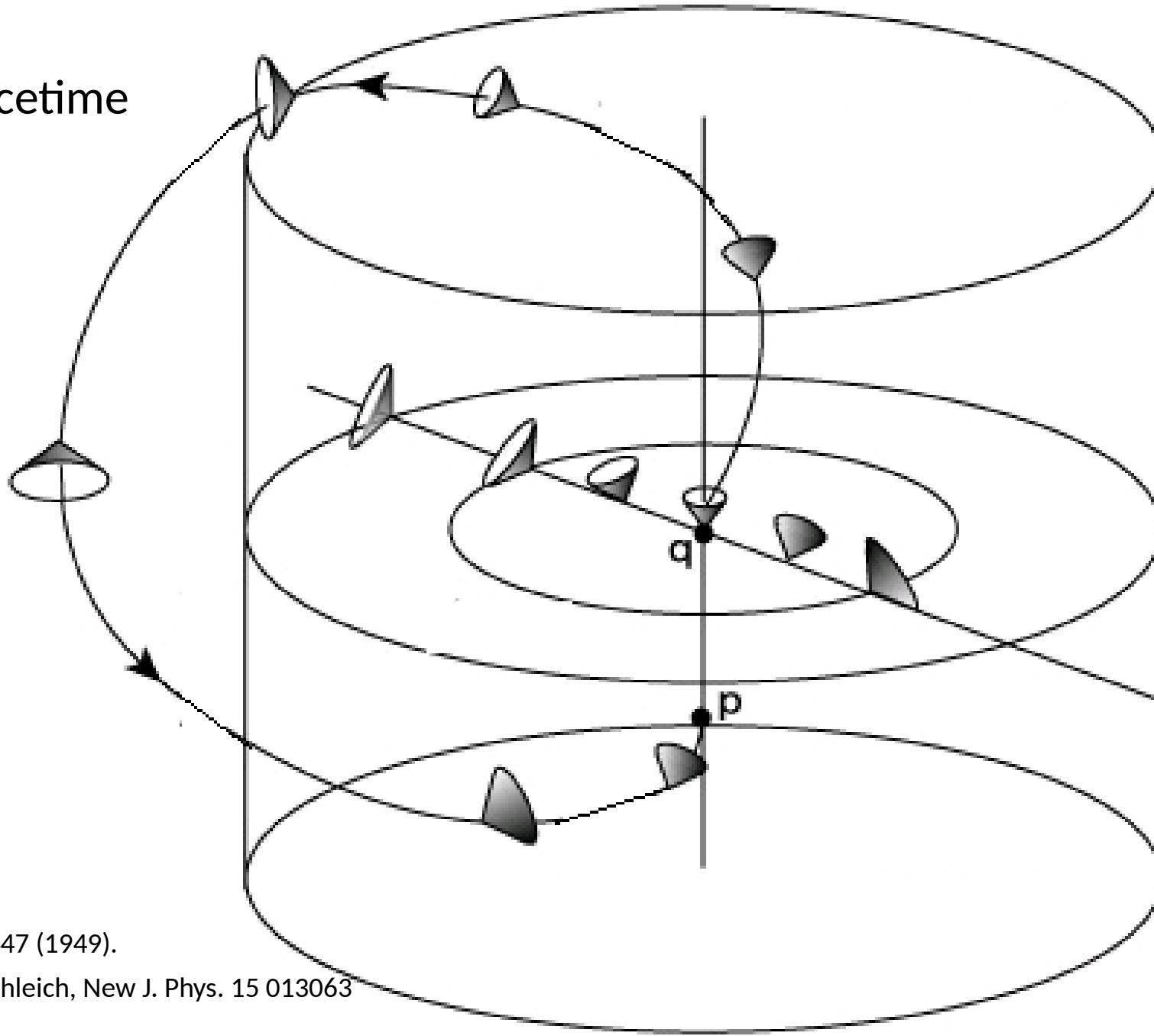


Time warp
for $t \in [0, \tau]$



$\{\psi(t): t\}$

Gödel spacetime



K. Gödel, *Rev. Mod. Phys.* **21** 447 (1949).

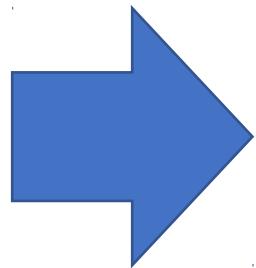
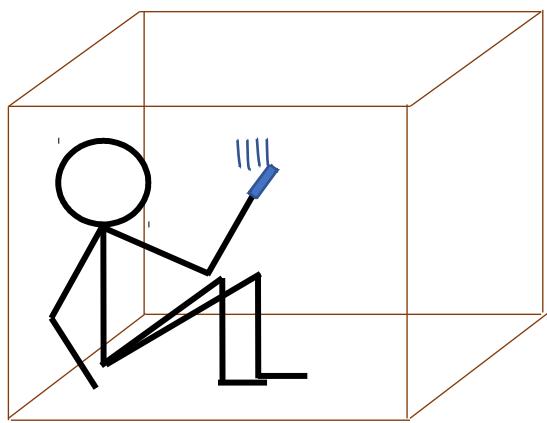
M. Buser, E. Kajari and W. P. Schleich, *New J. Phys.* **15** 013063 (2013).

Time travel with wormholes

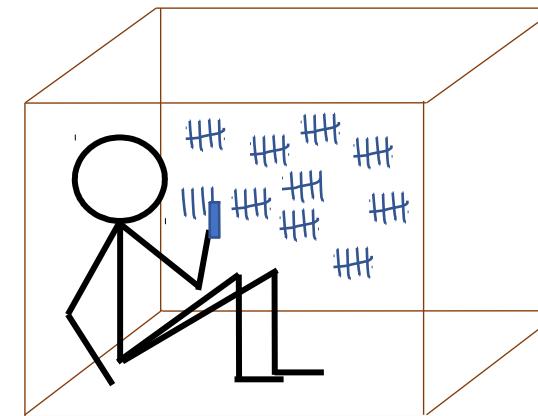


K. Thorne, Black Holes and Time Warps: Einstein's
Outrageous Legacy, Commonwealth Fund Book Program
(1994).

Time warp with time machines

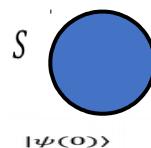


Time warp
for $t \in [0, \tau]$



$\{\psi(t): t\}$

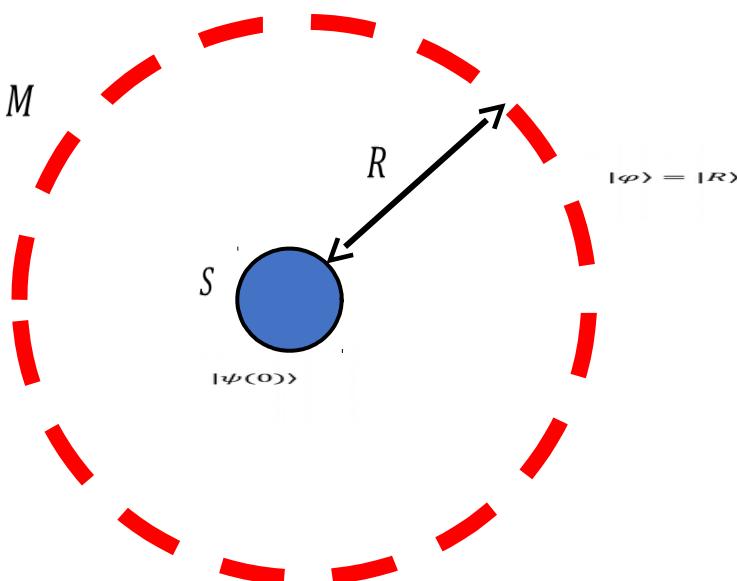
The time translator



Evolution after time T

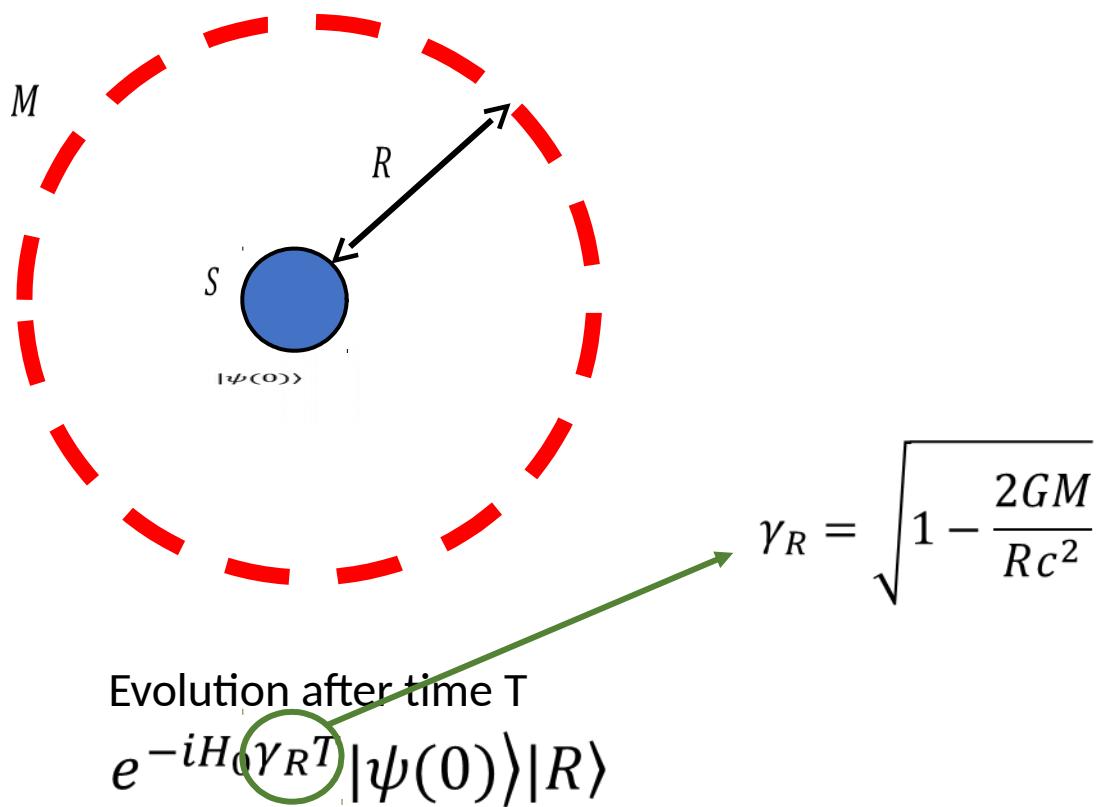
$$e^{-iH_0T} |\psi(0)\rangle$$

The time translator

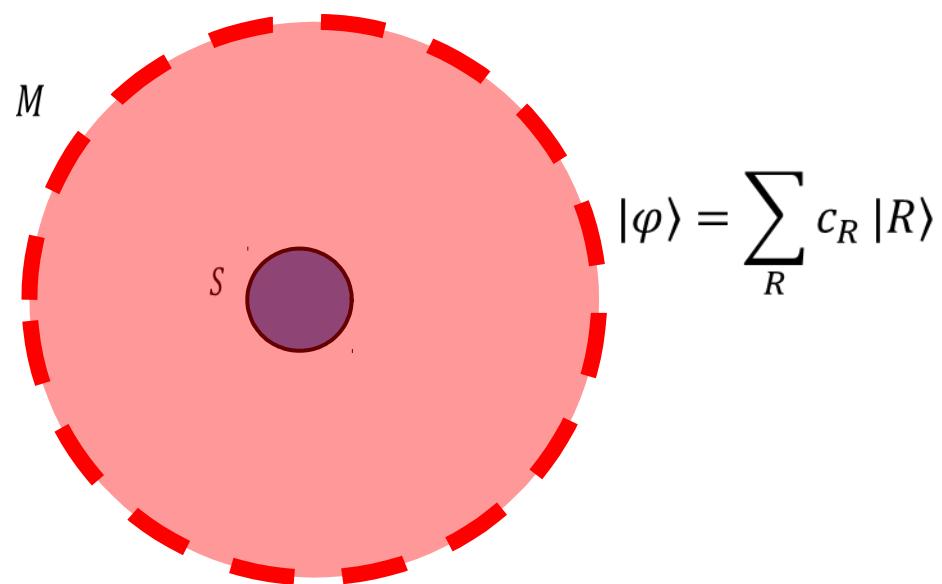


Evolution after time T
 $e^{-iH_0\gamma_R T} |\psi(0)\rangle |R\rangle$

The time translator



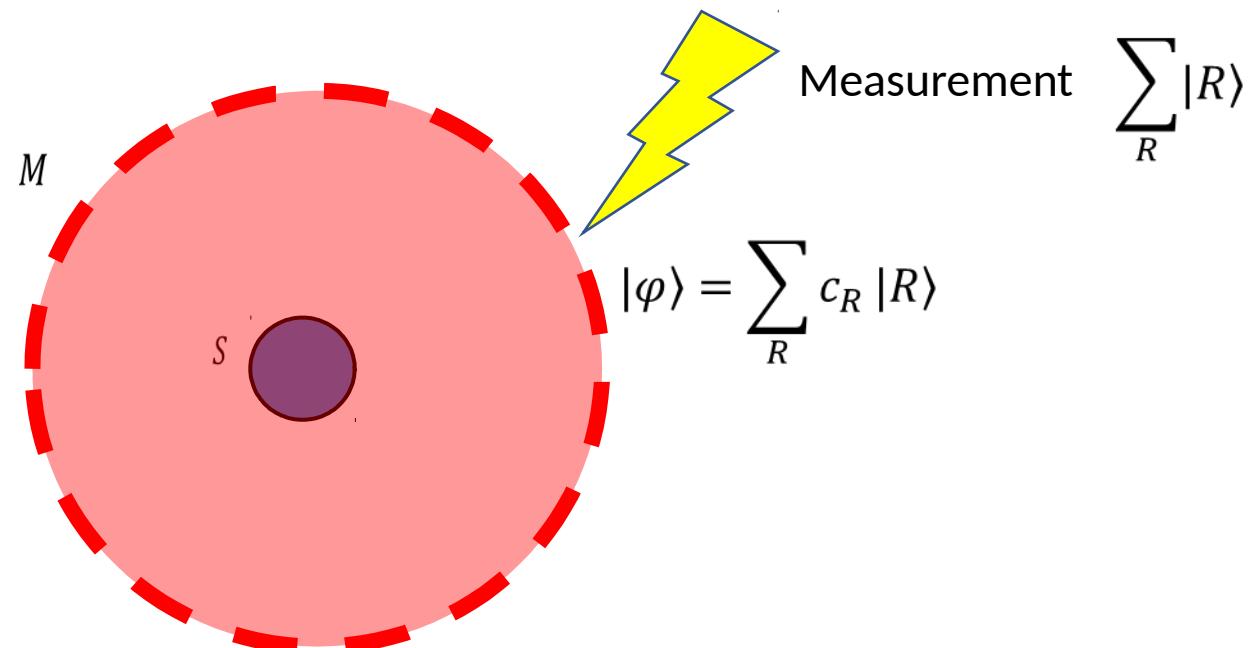
The time translator



Evolution after time T

$$\sum_R c_R e^{-iH_0\gamma_R T} |\psi(0)\rangle |R\rangle$$

The time translator

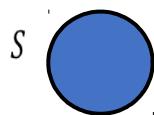


Evolution after time T

$$\sum_R c_R e^{-iH_0\gamma_R T} |\psi(0)\rangle |R\rangle$$

The time translator

M

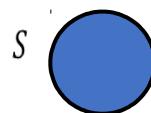


Evolution after time T

$$\sum_R c_R e^{-iH_0\gamma_R T} |\psi(0)\rangle$$

The time translator

M



Evolution after time $T \ll 1$

$$\sum_R c_R e^{-iH_0\gamma_R T} |\psi(0)\rangle \approx e^{-iH_0\alpha T} |\psi(0)\rangle$$

The time translator

M

S

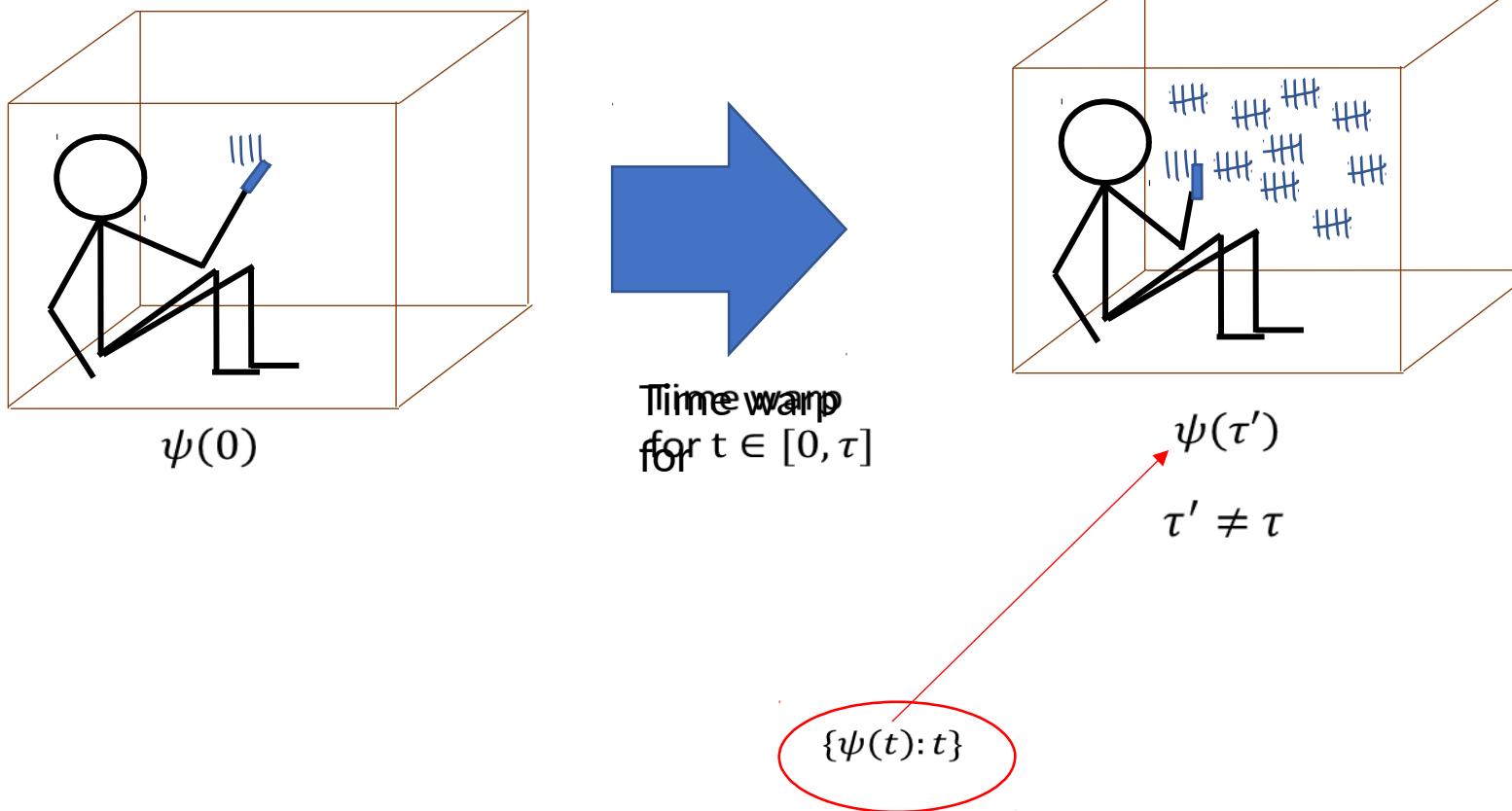
$\sum_R c_R e^{-iH_0\gamma_R T} |\psi(0)\rangle \approx e^{-iH_0\alpha T} |\psi(0)\rangle$

Evolution after time $T \ll 1$

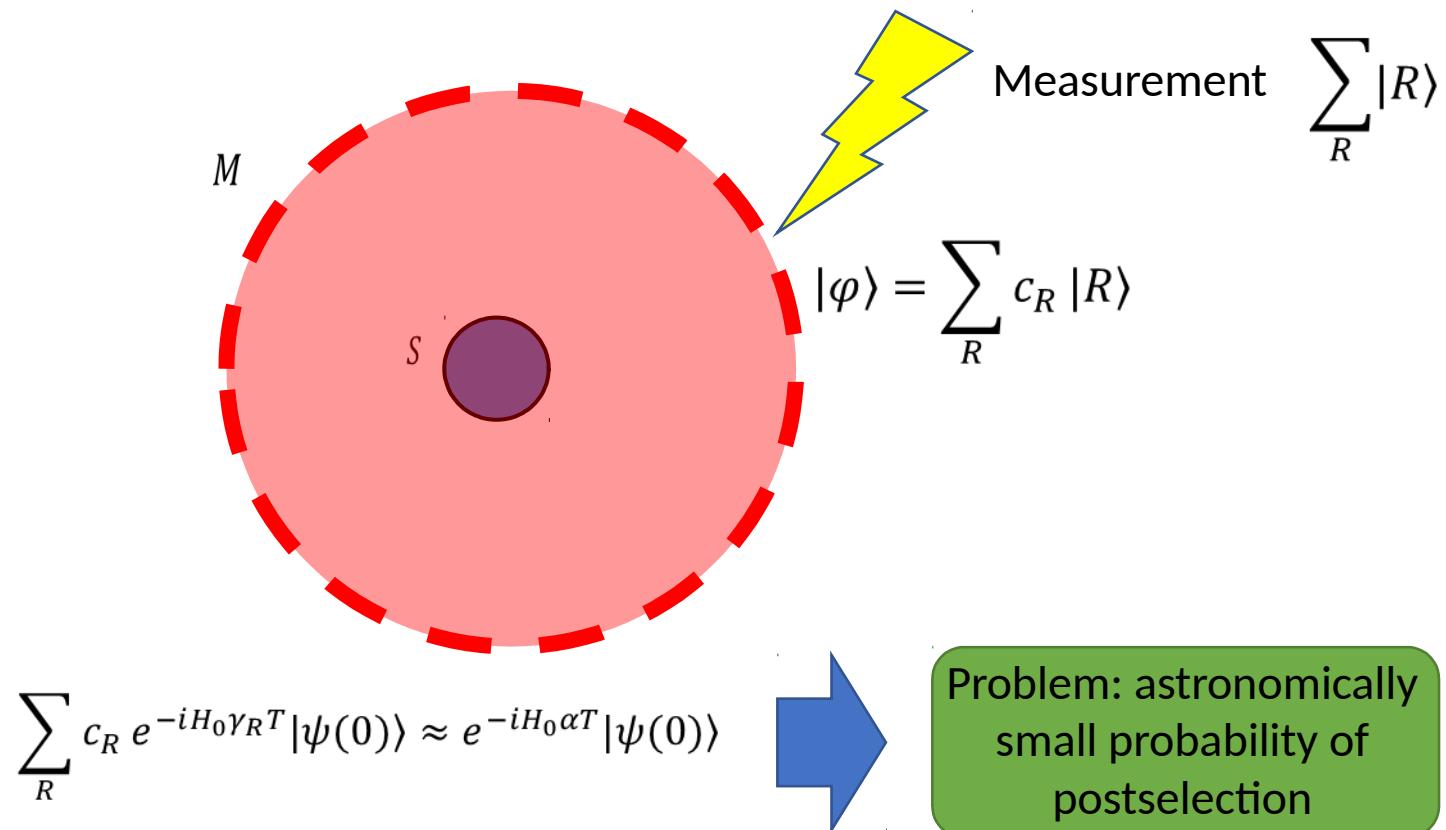
Interesting cases $\left\{ \begin{array}{l} \alpha \gg 1 \\ \alpha \leq 0 \end{array} \right.$

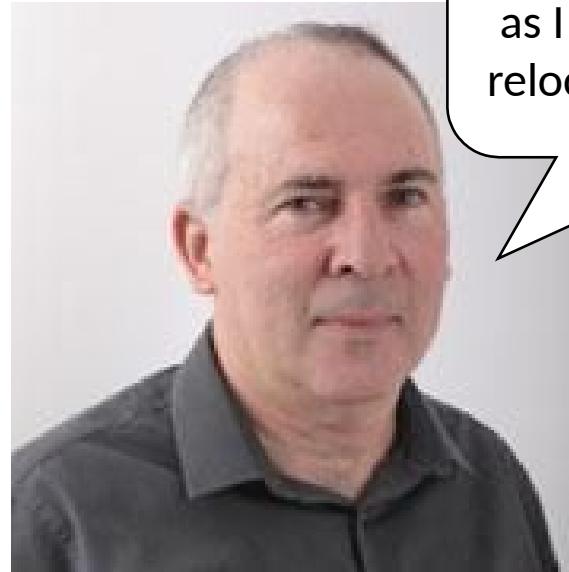
A diagram consisting of a blue circle labeled 'S' positioned above a mathematical expression. The expression is a sum over R of coefficients c_R multiplied by a term involving a Hamiltonian H_0 , a parameter γ_R , and time T . Below this, the text 'Evolution after time $T \ll 1$ ' is written. To the right, a green arrow points towards a brace that defines 'Interesting cases' for the parameter α , with two options: $\alpha \gg 1$ and $\alpha \leq 0$.

Time warp with the time translator



The time translator

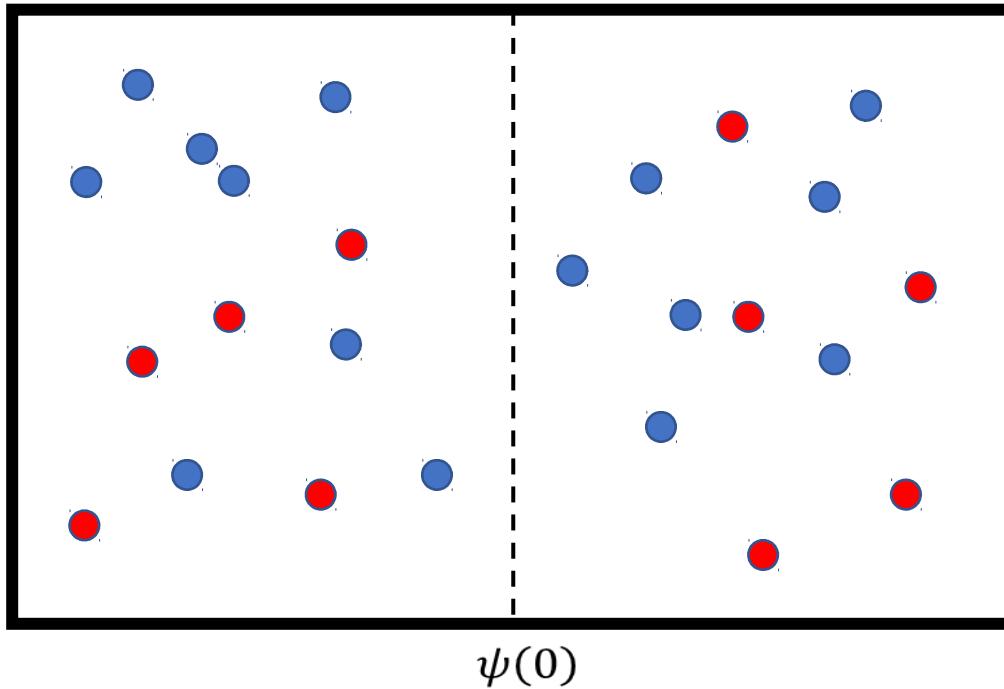




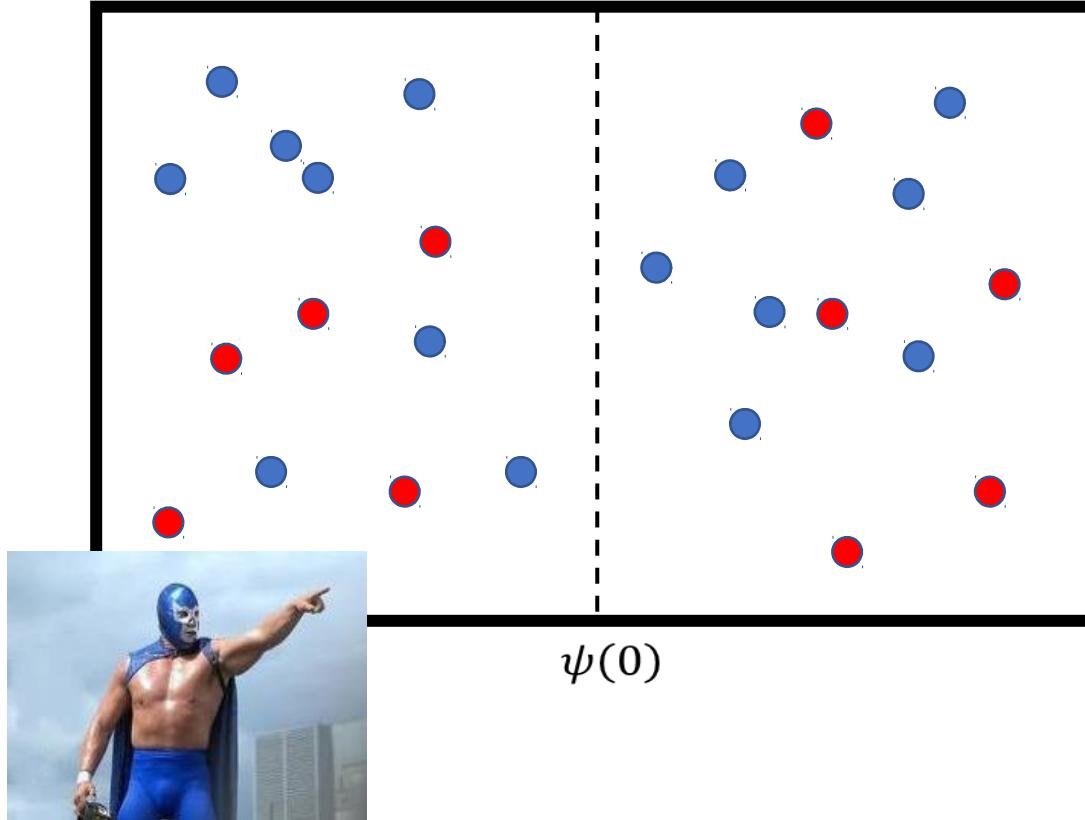
“The time translator has the same chances of succeeding as I have of delocalizing and relocalizing somewhere else”

L. Vaidman

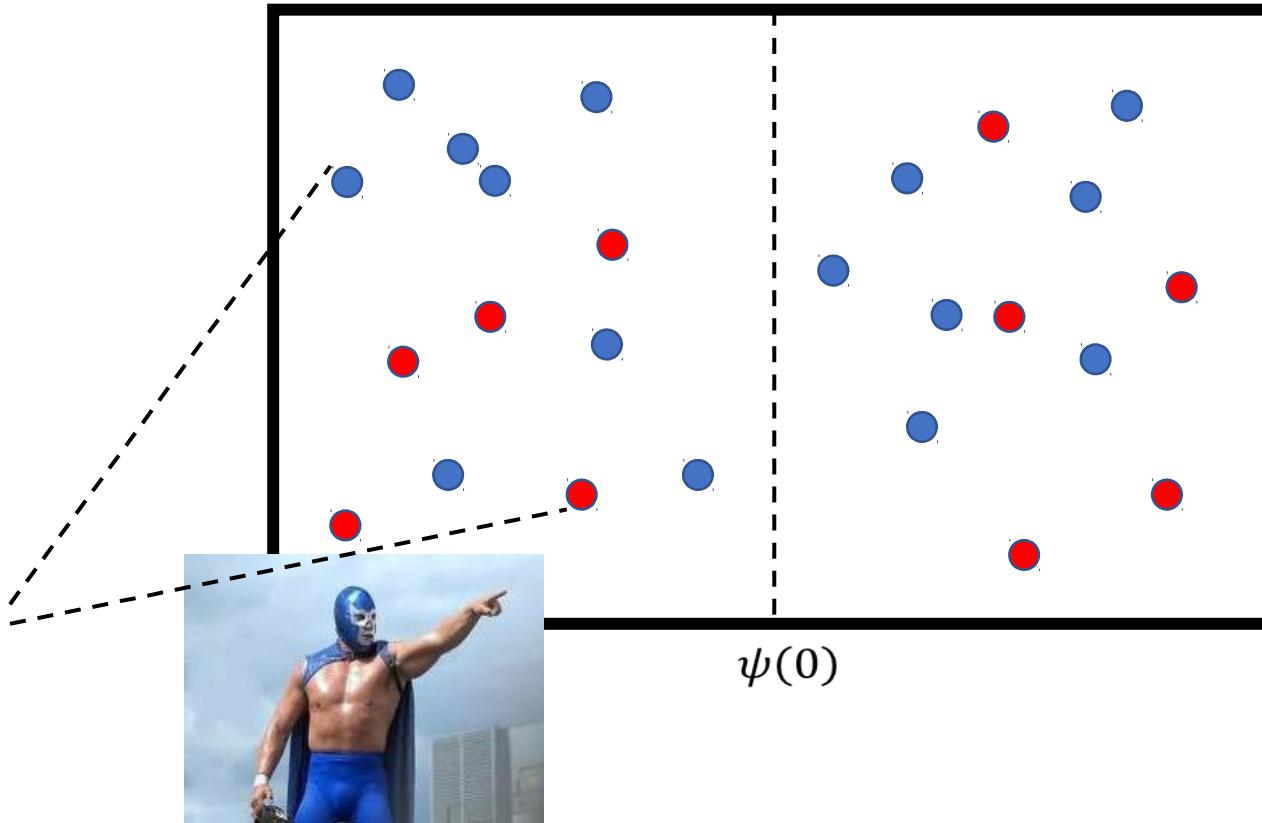
Time warp, the lame way



Time warp, the lame way

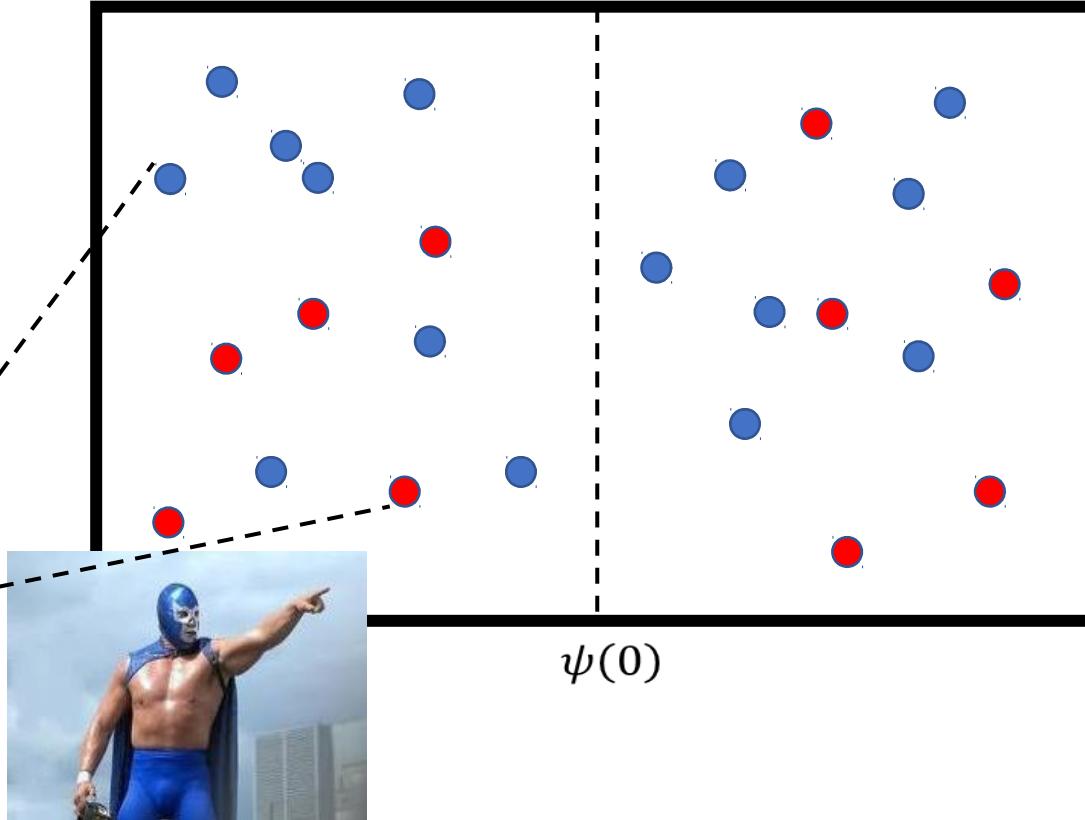


Time warp, the lame way

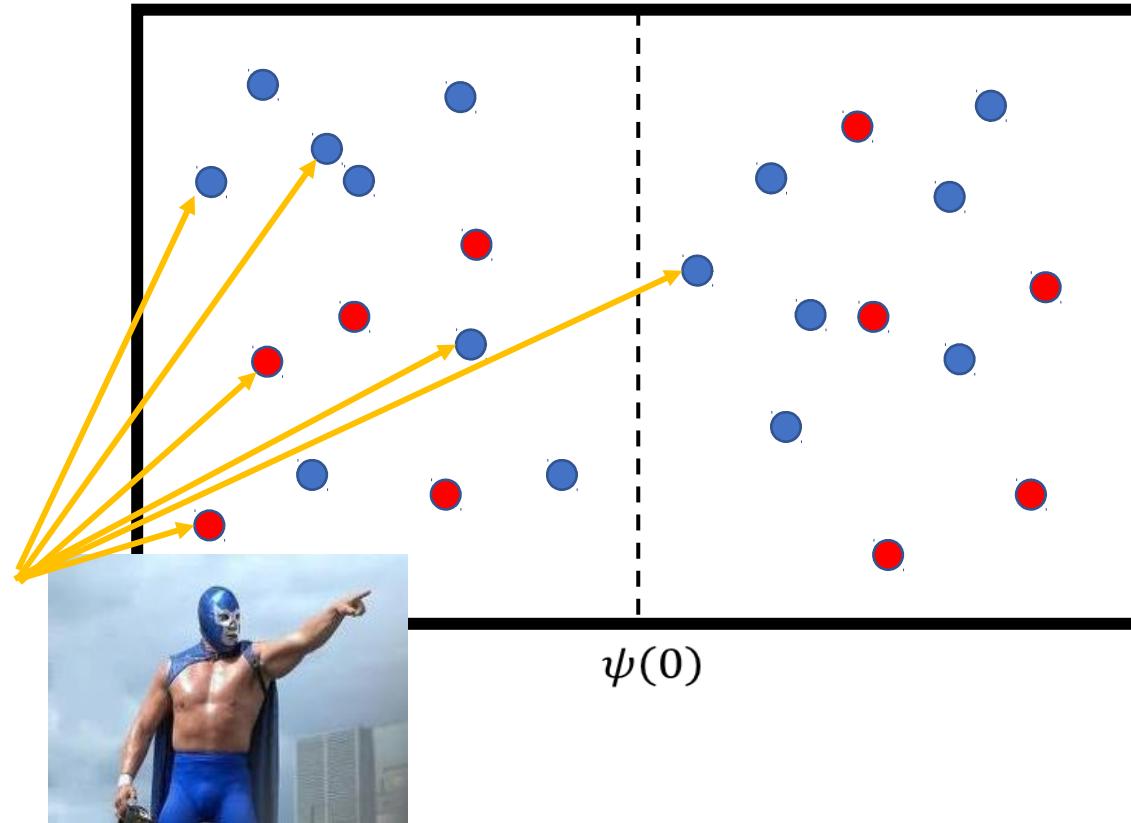


Time warp, the lame way

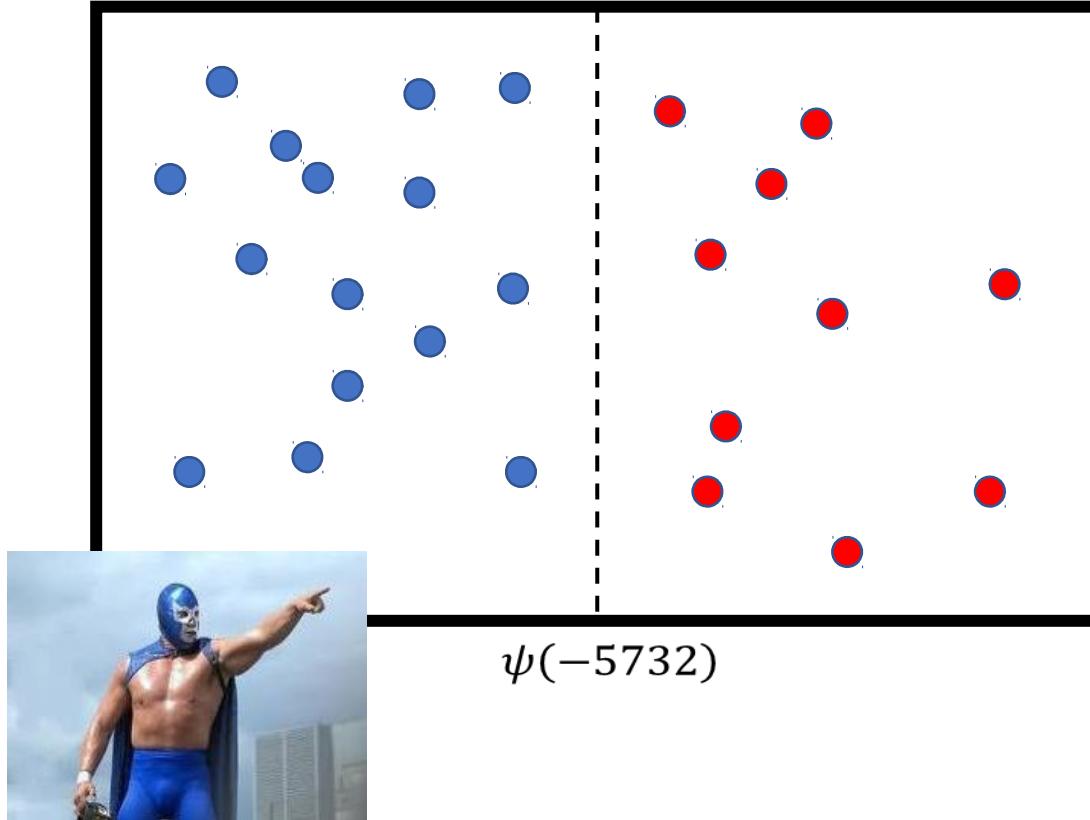
(0.3,0.2,0.9),
(0.4,0.7,0.1),
(0.5,0.6,0.3),

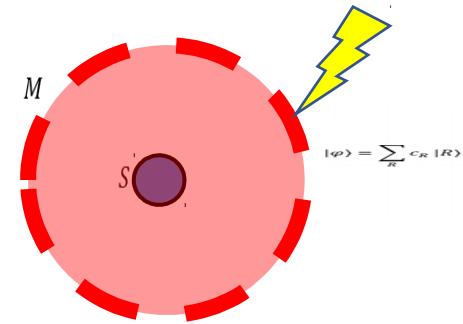
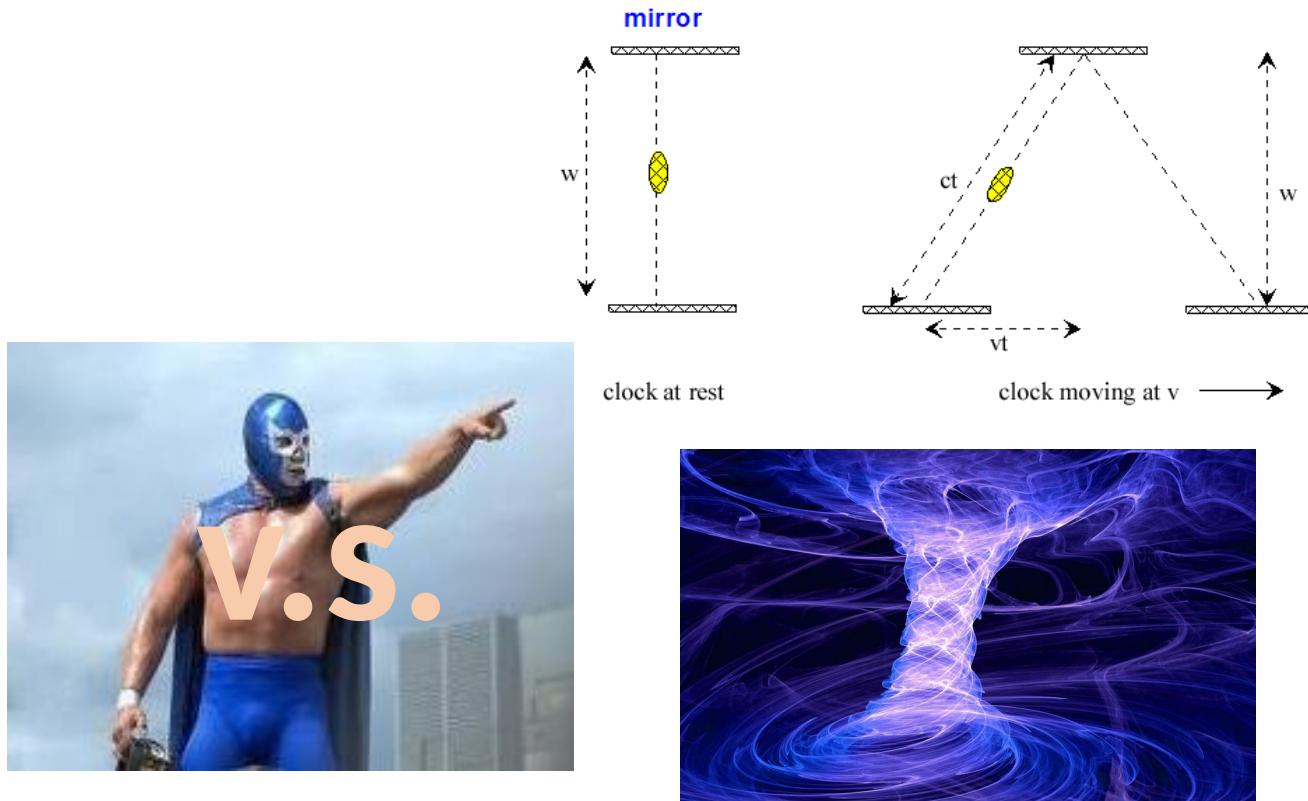


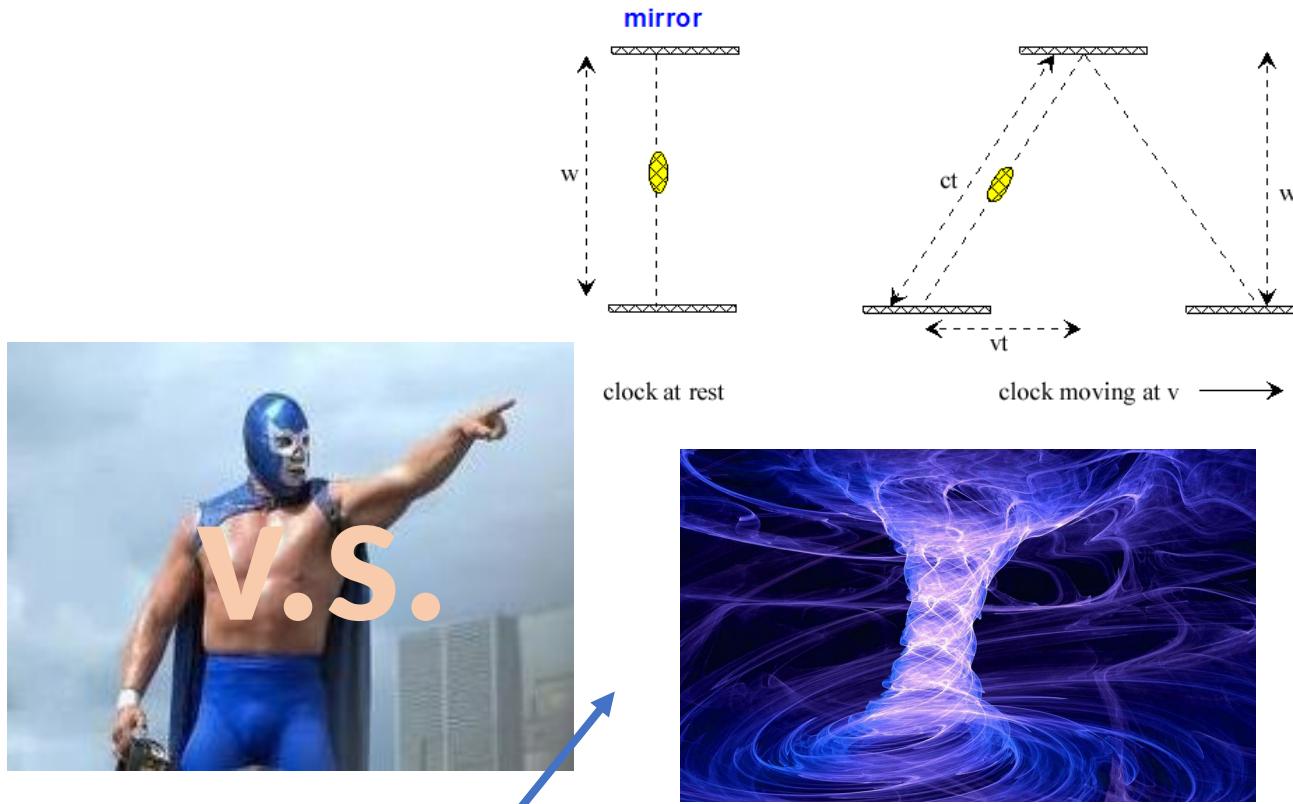
Time warp, the lame way



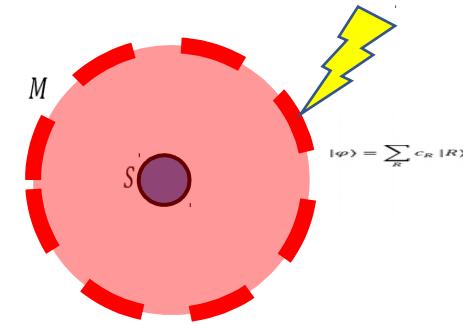
Time warp, the lame way

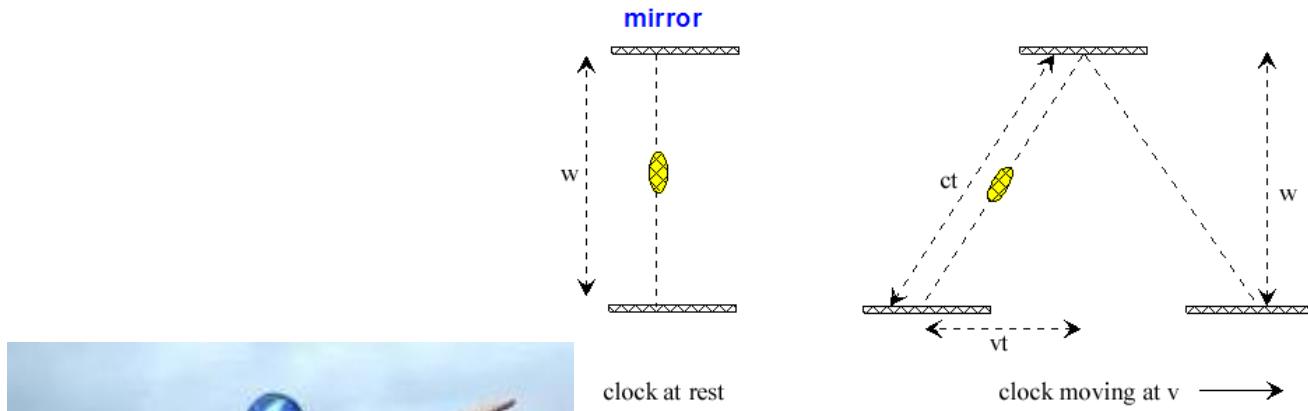




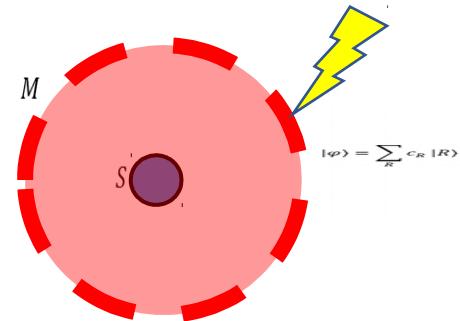


Do not require control or knowledge of the physical system that we influence

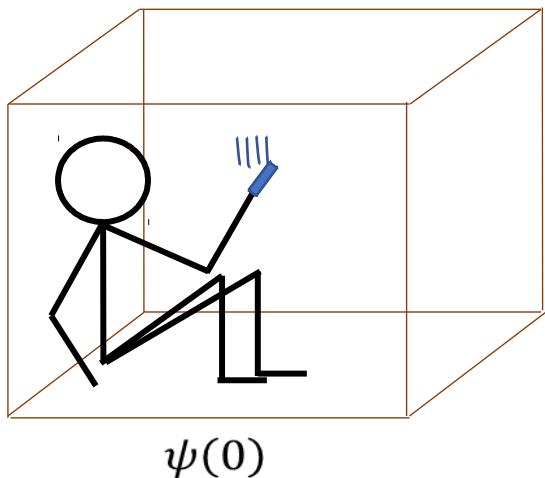




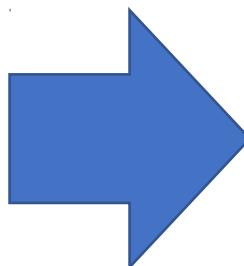
Rely on special or general relativity



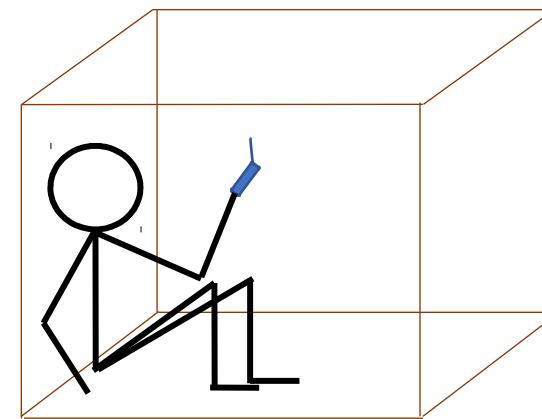
Main result



Uncontrolled system

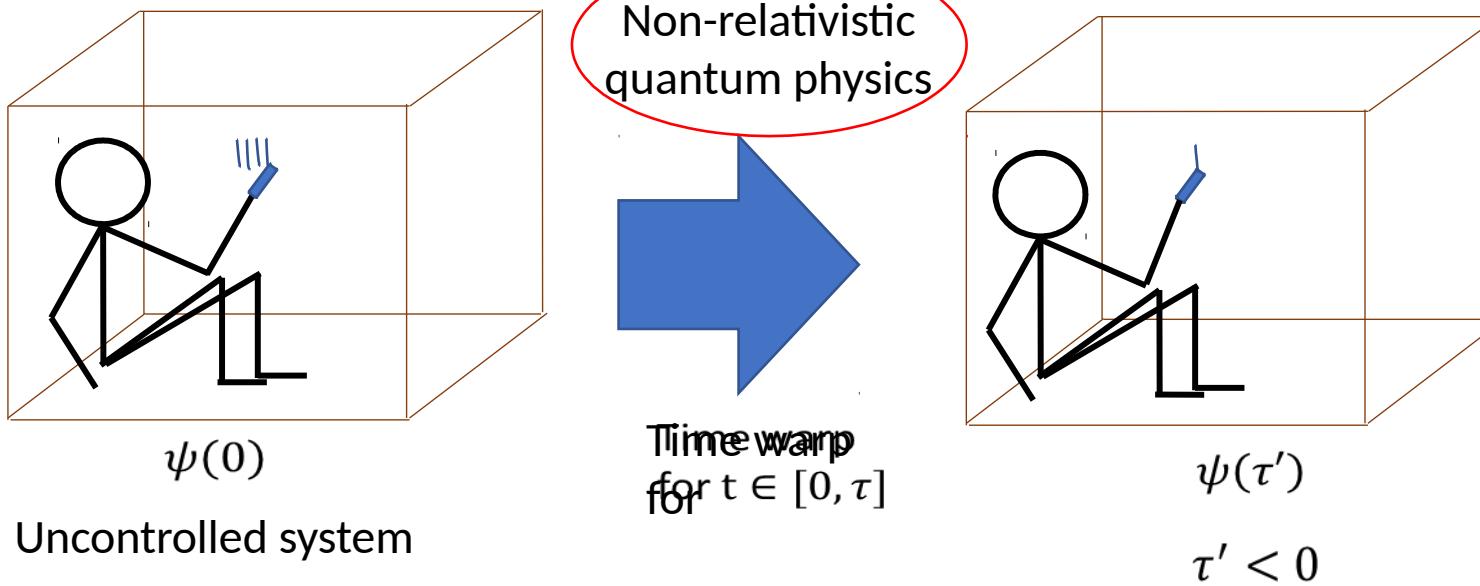


Time warp
for $t \in [0, \tau]$

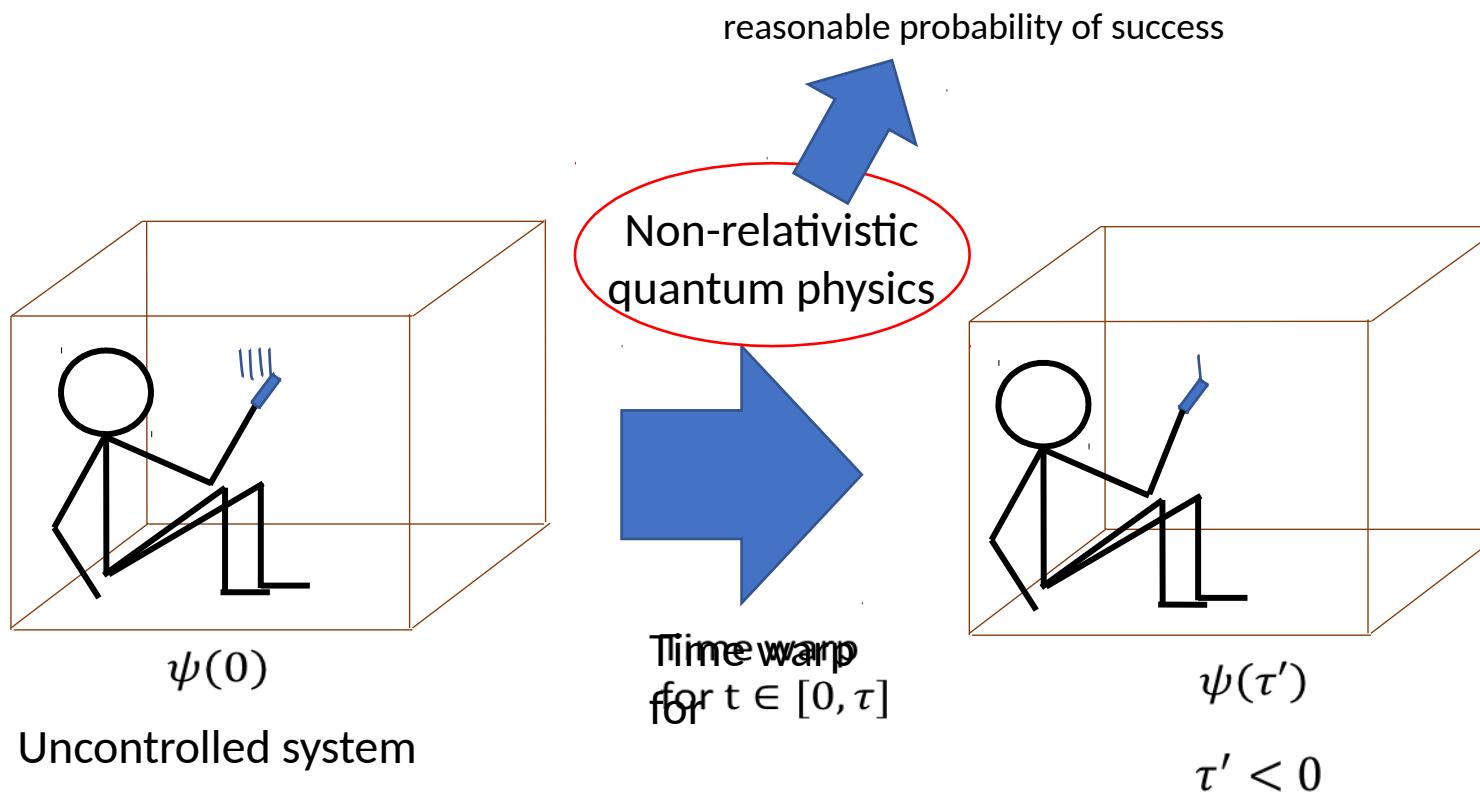


$\tau' < 0$

Main result



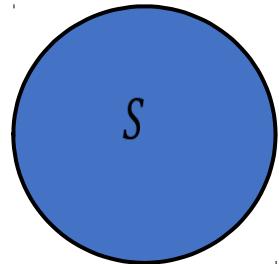
Main result



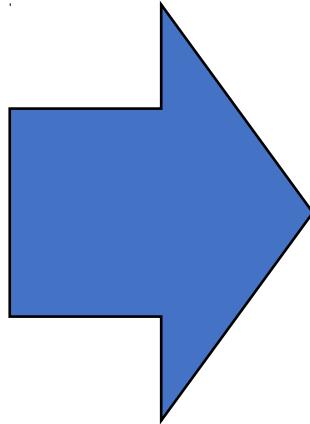
Scenario

Goal

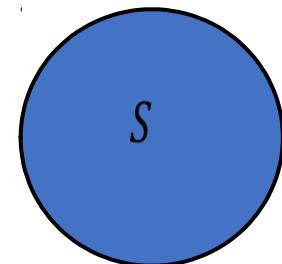
$$|\psi(T)\rangle = e^{-iH_0T}|\psi(0)\rangle$$



$$t = T > 0$$



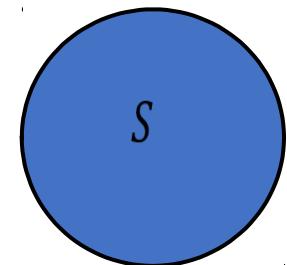
$$|\psi(0)\rangle$$



$$t = T + \Delta$$

Obvious solution

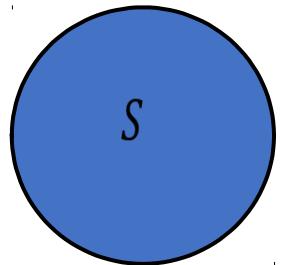
$$|\psi(T)\rangle = e^{-iH_0T}|\psi(0)\rangle$$



$$|\psi(0)\rangle = e^{+iH_0T}|\psi(T)\rangle$$

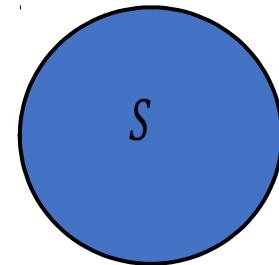
Obvious solution

$$|\psi(T)\rangle = e^{-iH_0T}|\psi(0)\rangle$$



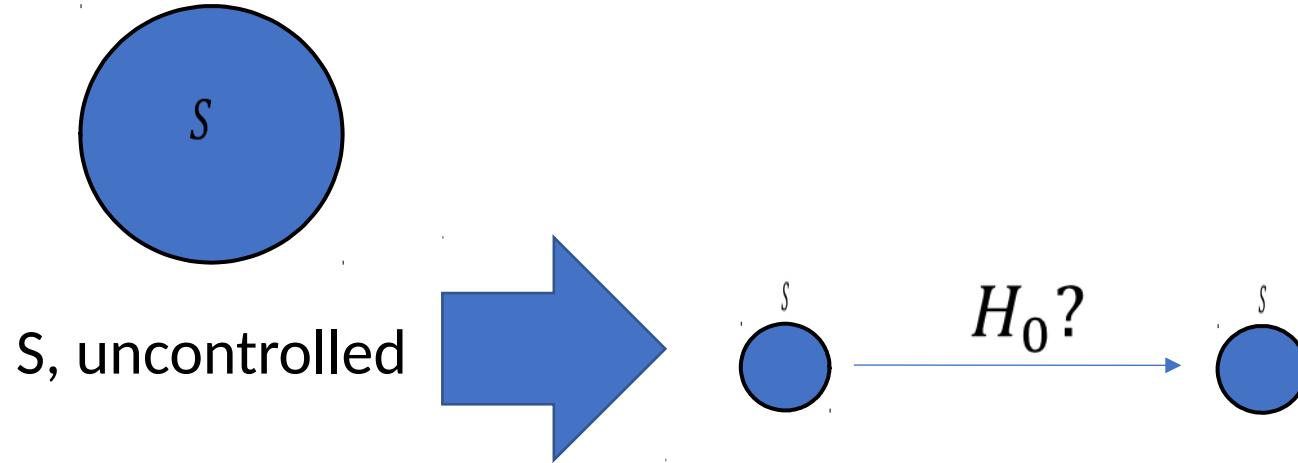
$$|\psi(0)\rangle = e^{+iH_0T}|\psi(T)\rangle$$

$$|\psi(T)\rangle = e^{-iH_0T}|\psi(0)\rangle$$

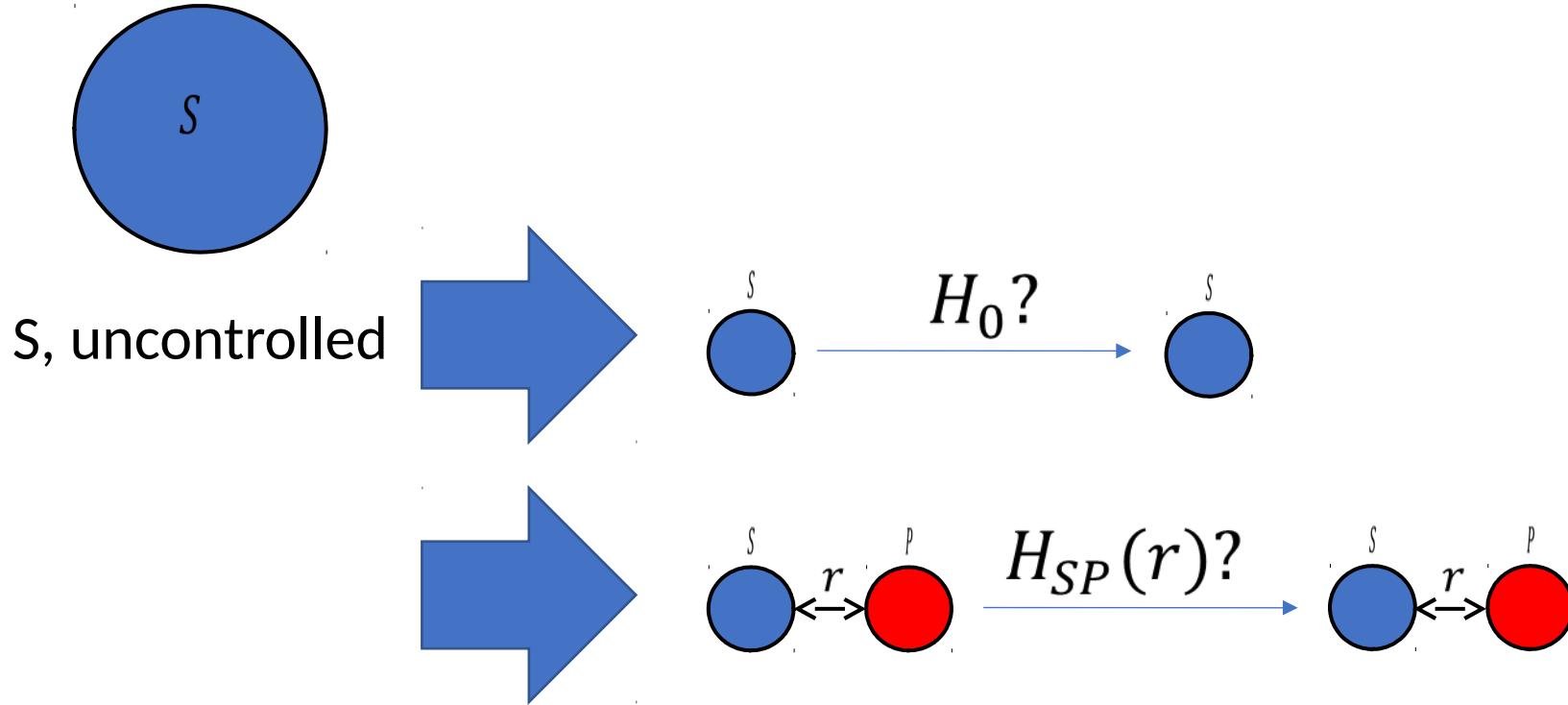


S, uncontrolled

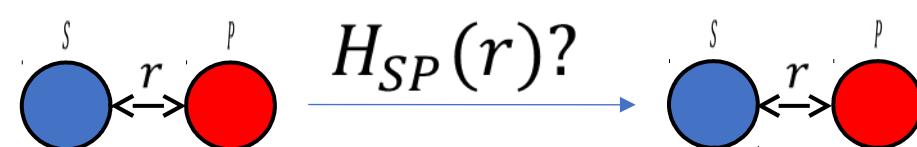
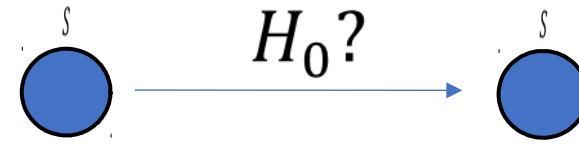
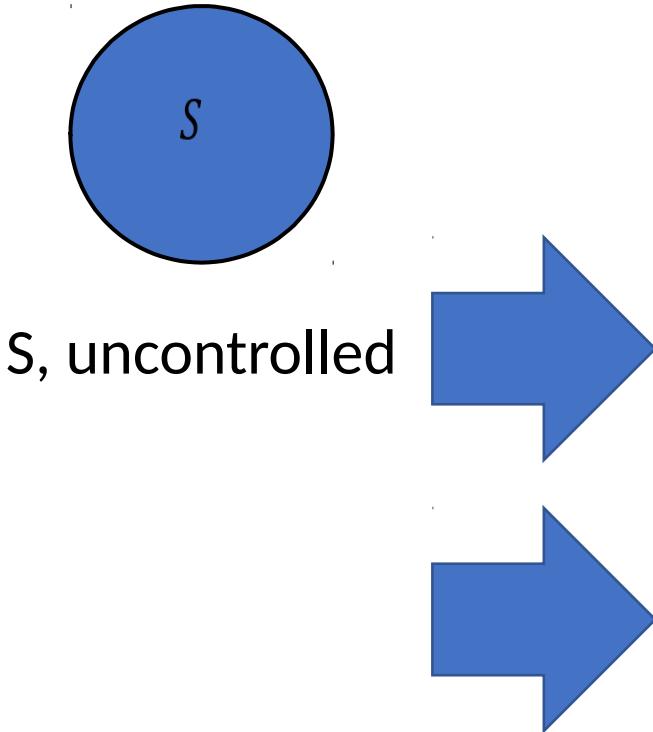
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



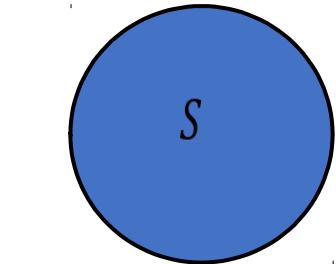
$$|\psi(T)\rangle = e^{-iH_0 T} |\psi(0)\rangle$$



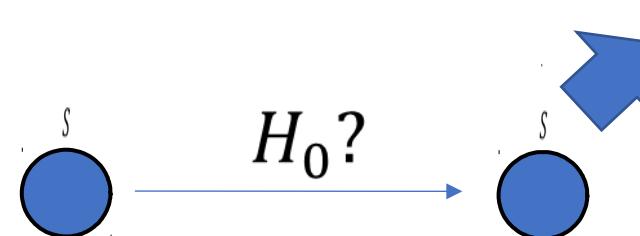
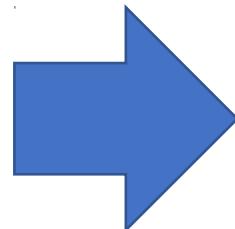
$$|\psi(0)\rangle = e^{+iH_0 T} |\psi(T)\rangle ?$$

A large red diagonal slash is drawn through the text above.

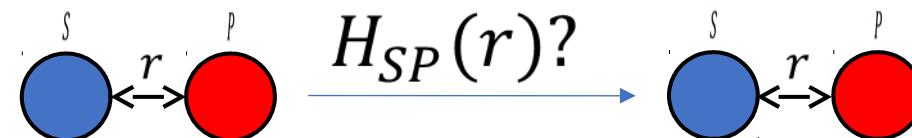
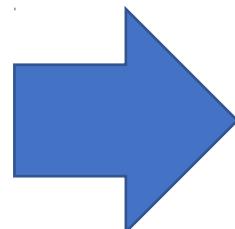
$$|\psi(T)\rangle = e^{-iH_0 T} |\psi(0)\rangle$$



S, uncontrolled

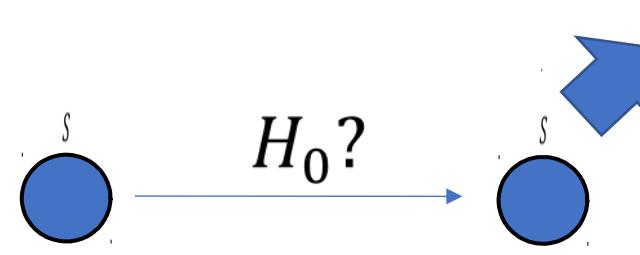
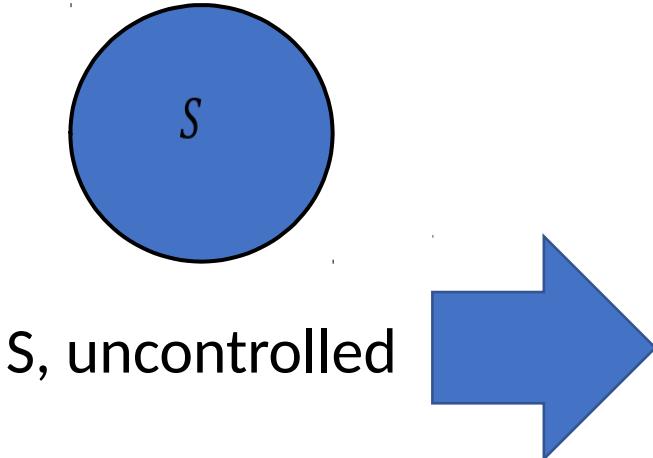


We don't know H_0 .



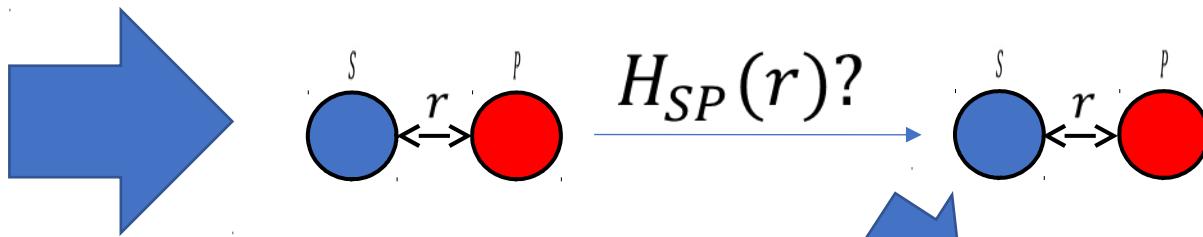
~~$$|\psi(0)\rangle = e^{+iH_0 T} |\psi(T)\rangle?$$~~

$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$

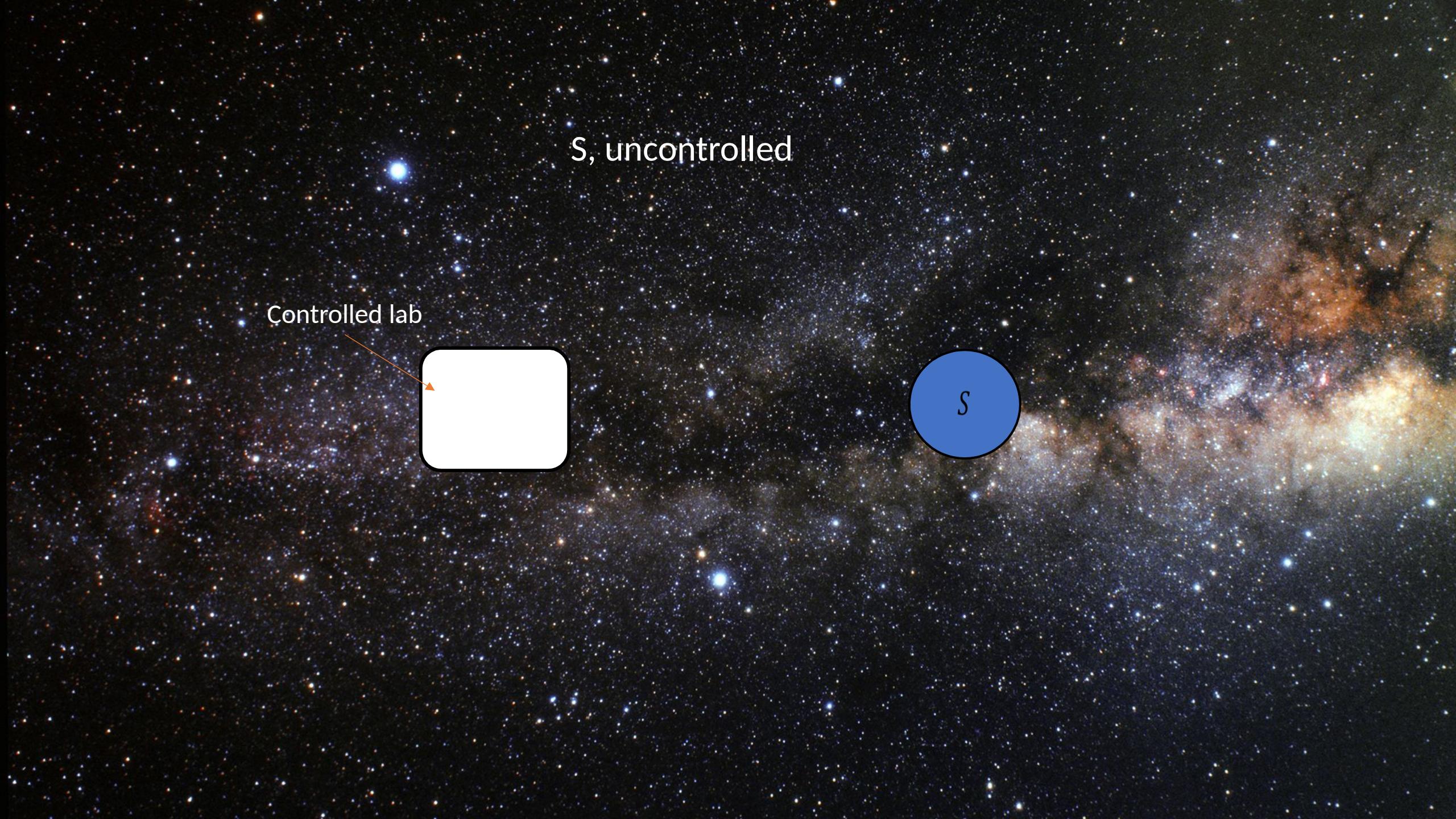


~~$|\psi(0)\rangle = e^{+iH_0T} |\psi(T)\rangle?$~~

We don't know H_0 .



Even if we knew H_0 , we wouldn't know how to implement e^{+iH_0T} on S .

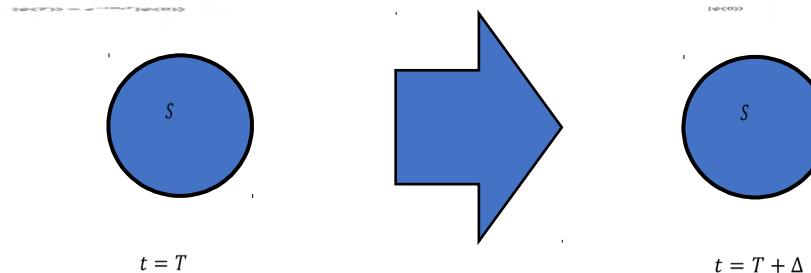


S, uncontrolled

Controlled lab

S

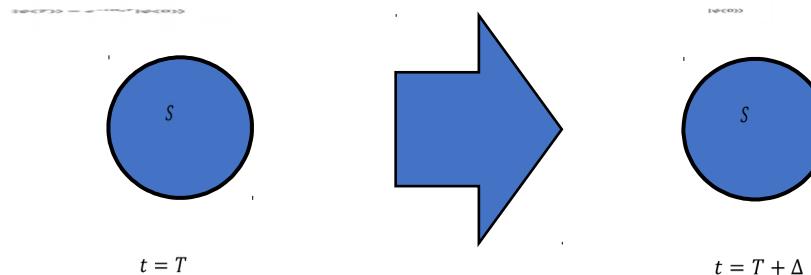
Resetting



We ignore how S evolves (unitarily) by itself and with other quantum systems

We know $\dim(H_s)$

Resetting



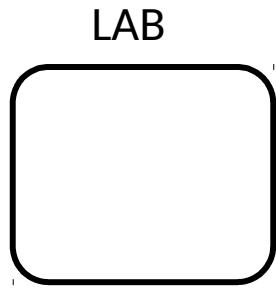
We ignore how S evolves (unitarily) by itself and with other quantum systems

We know $\dim(H_S)$

Impossible if we drop any
of the two assumptions

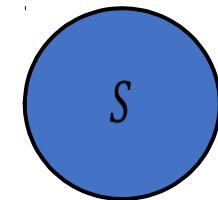
Sketch of a quantum resetting protocol

Initial conditions

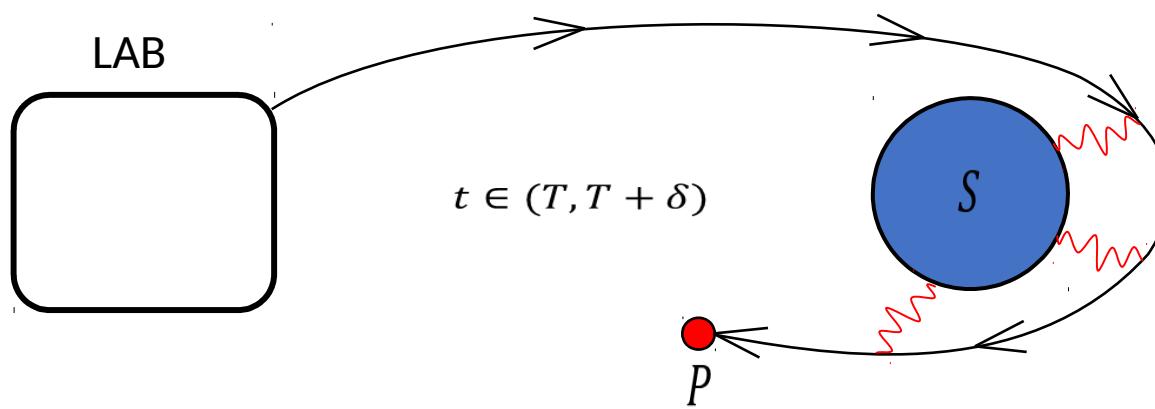


$t = T$

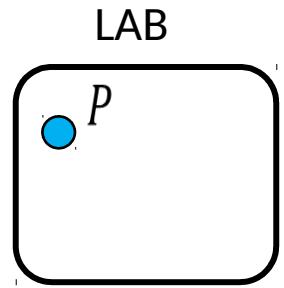
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



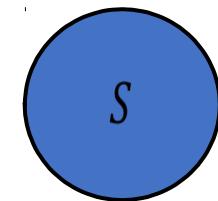
(a) Probe interaction



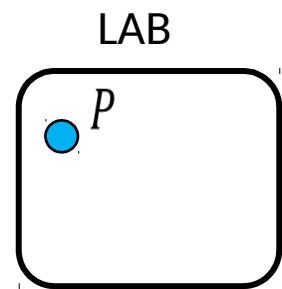
(a) Probe interaction



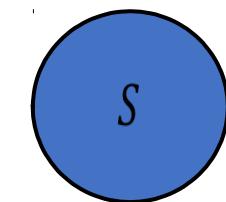
$t = T + \delta$



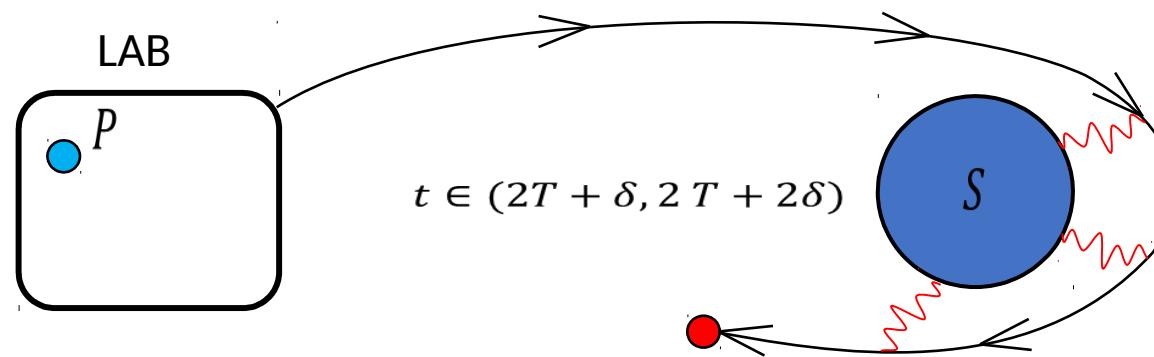
(b) Rest



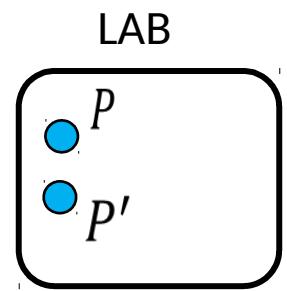
$t \in (T + \delta, 2T + \delta)$



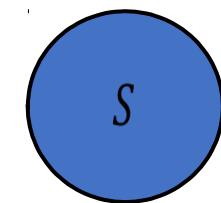
(a) Probe interaction



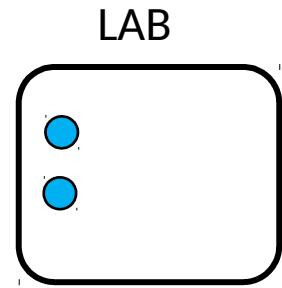
(a) Probe interaction



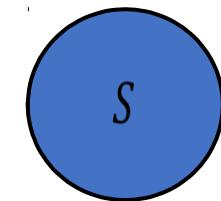
$$t = 2(T + \delta)$$



(b) Rest

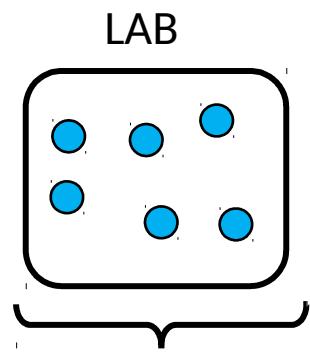


$t \in (2T + 2\delta, 3T + 2\delta)$



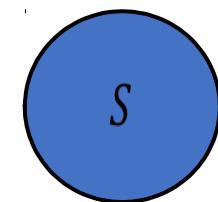


(a) Probe interaction

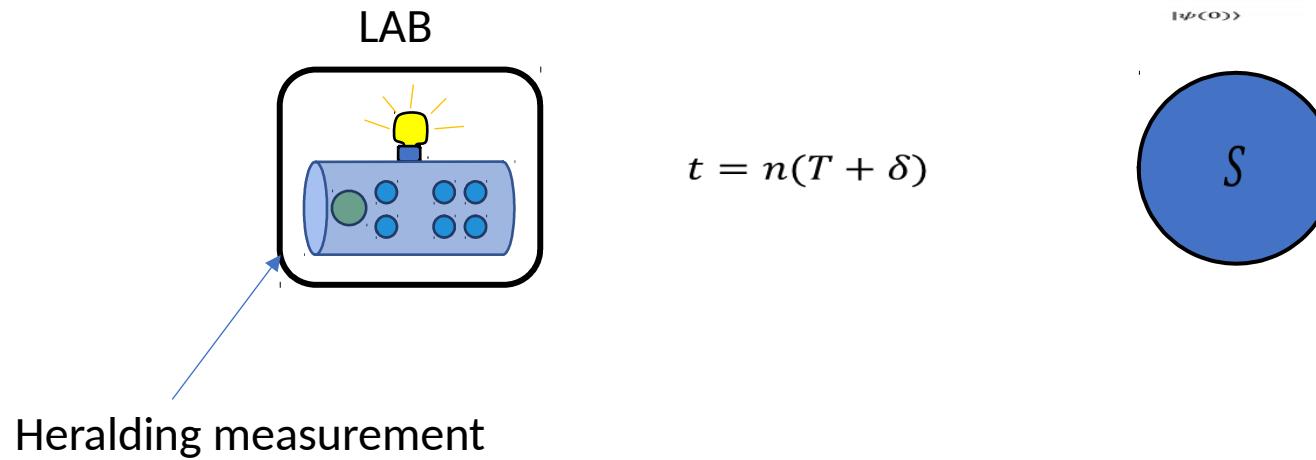


$n_{\text{returned}}^{\text{probes}}$

$$t = n(T + \delta)$$

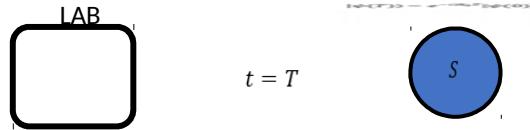


(c) Probe processing

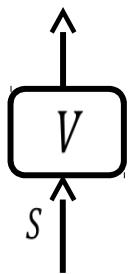


In the language of process diagrams

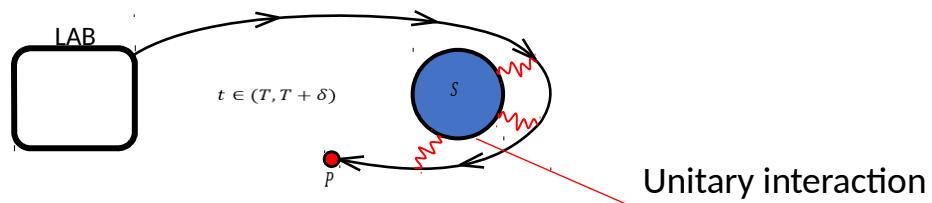
Initial conditions



$$V = e^{-iH_0T}$$

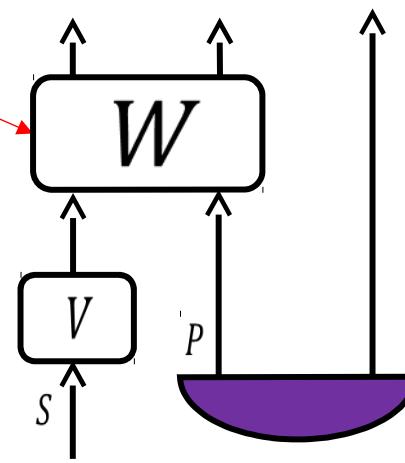


(a) Probe interaction

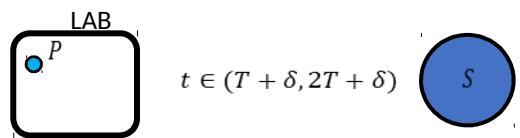


Unitary interaction

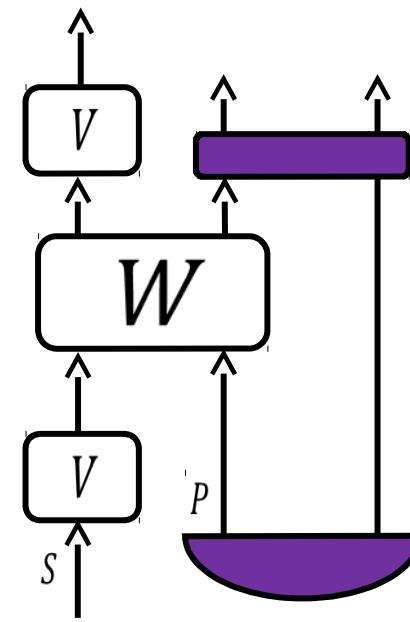
$$V = e^{-iH_0T}$$



(b) Rest

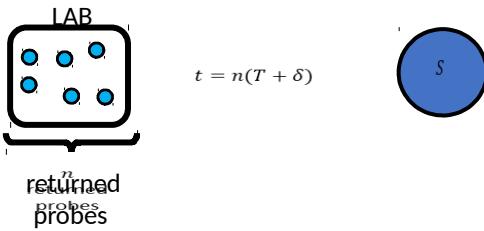


$$V = e^{-iH_0 T}$$

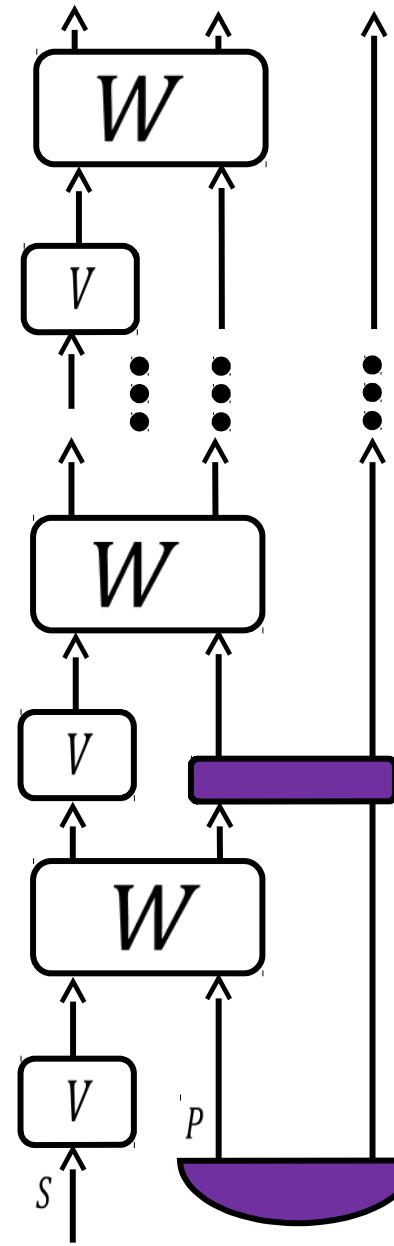




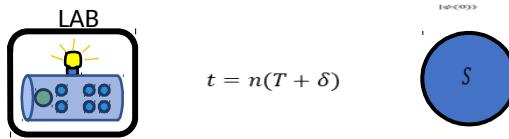
(a) Probe interaction



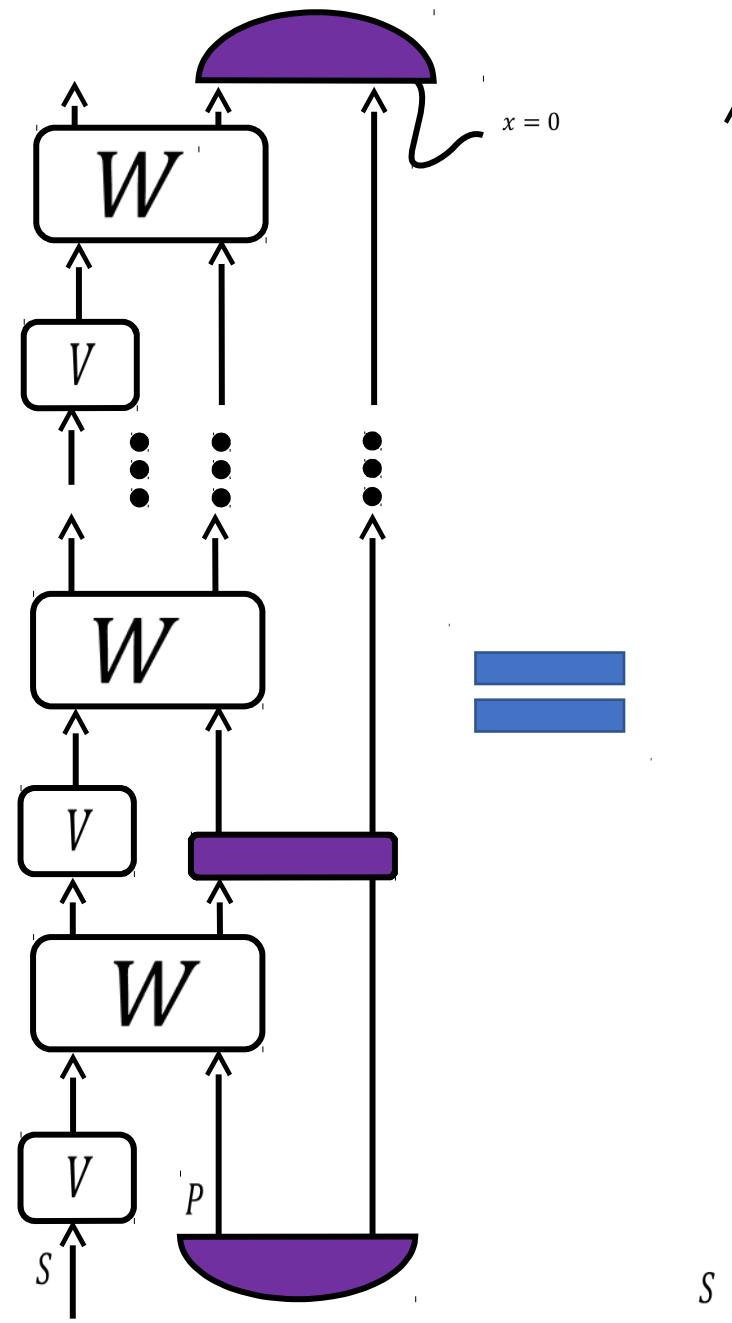
$$V = e^{-iH_0T}$$

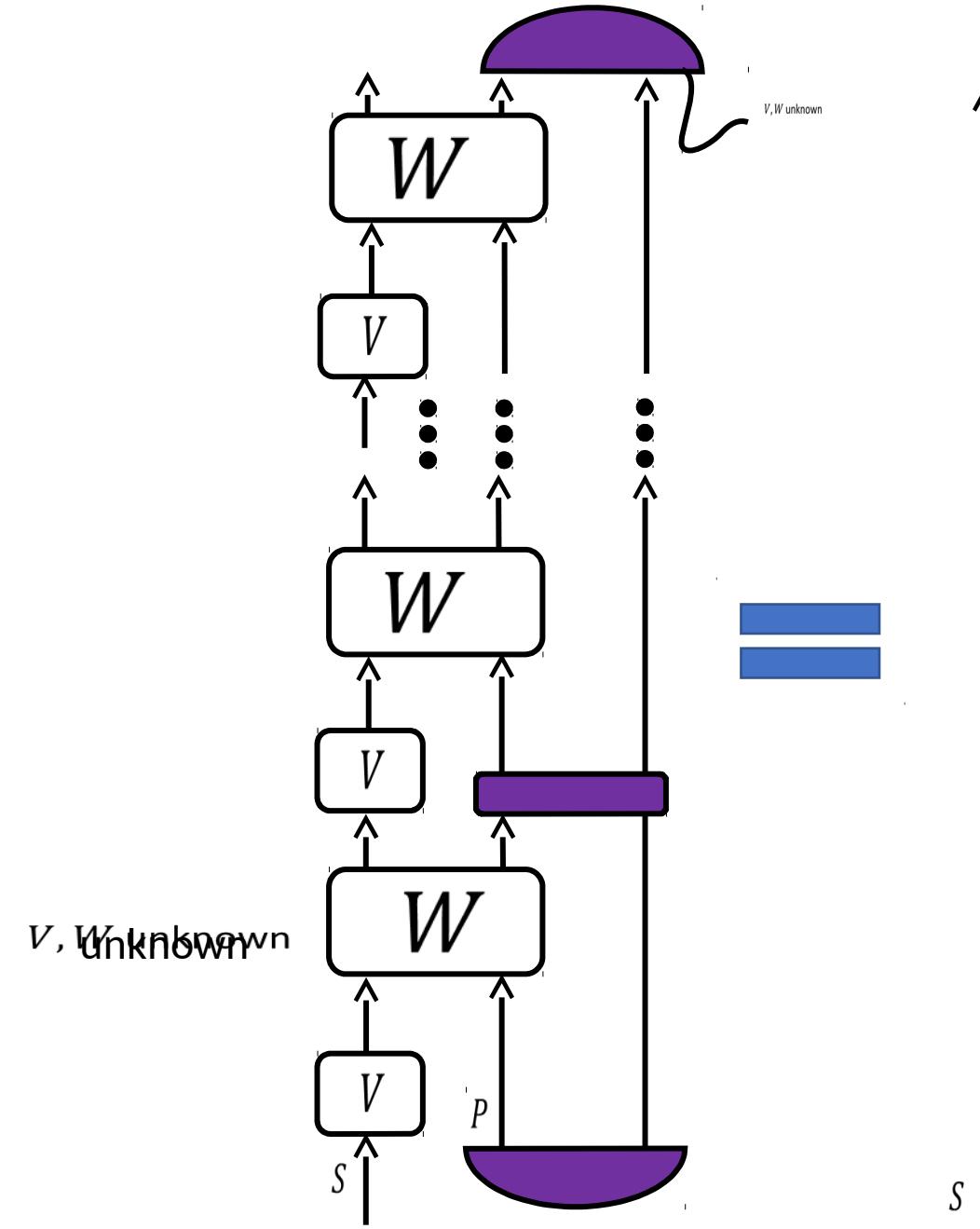


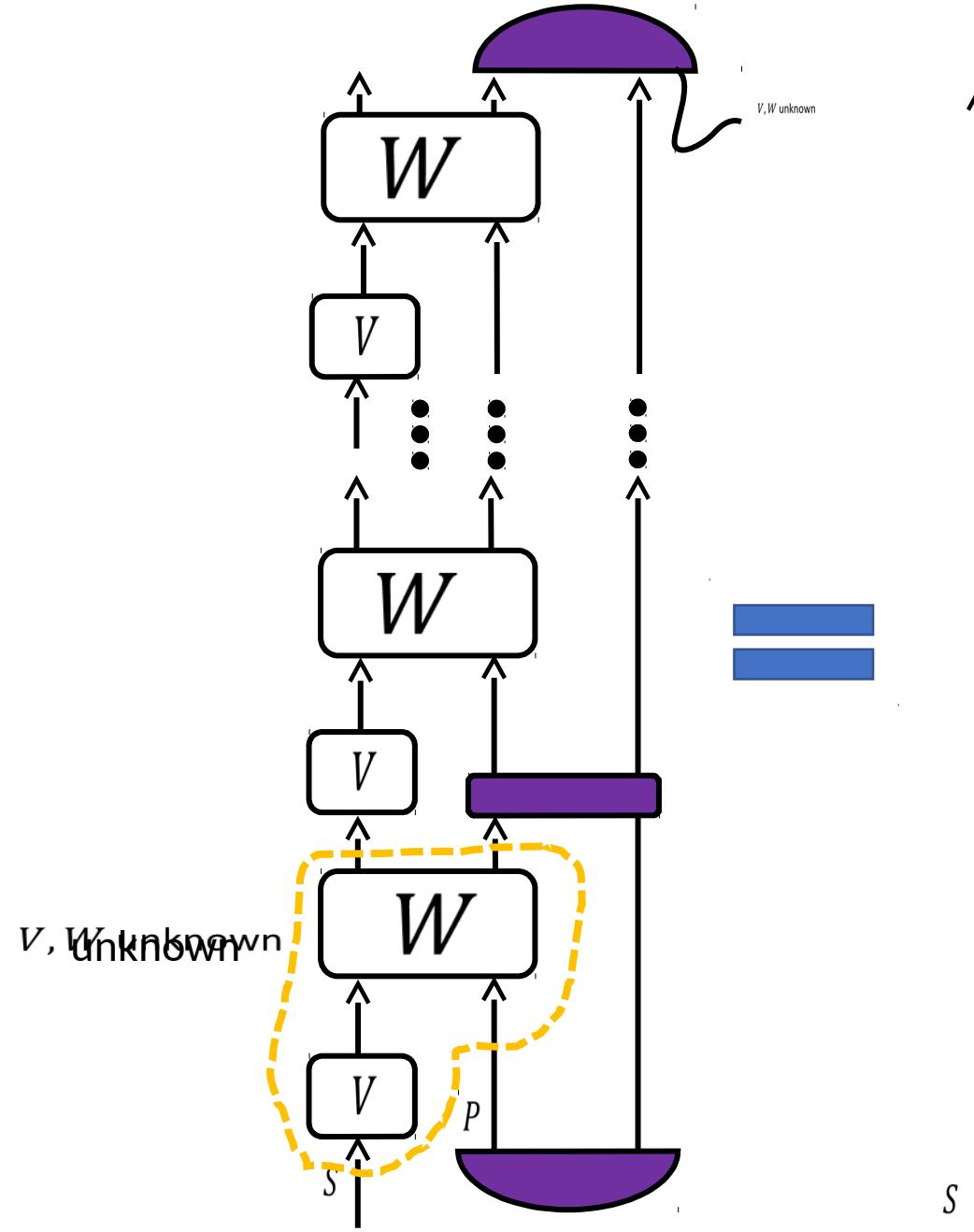
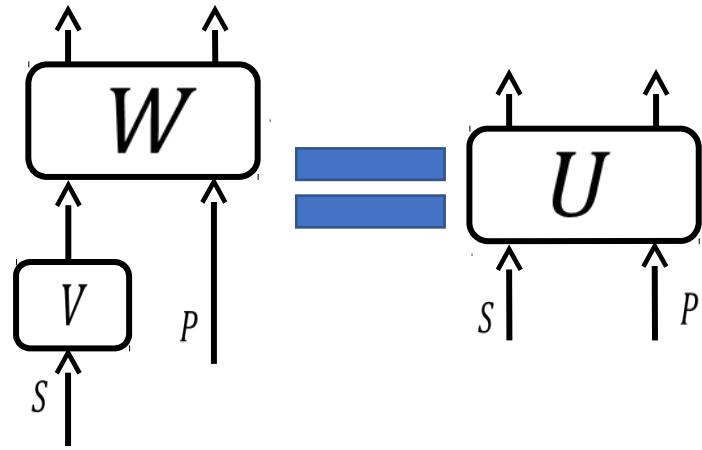
(c) Probe processing



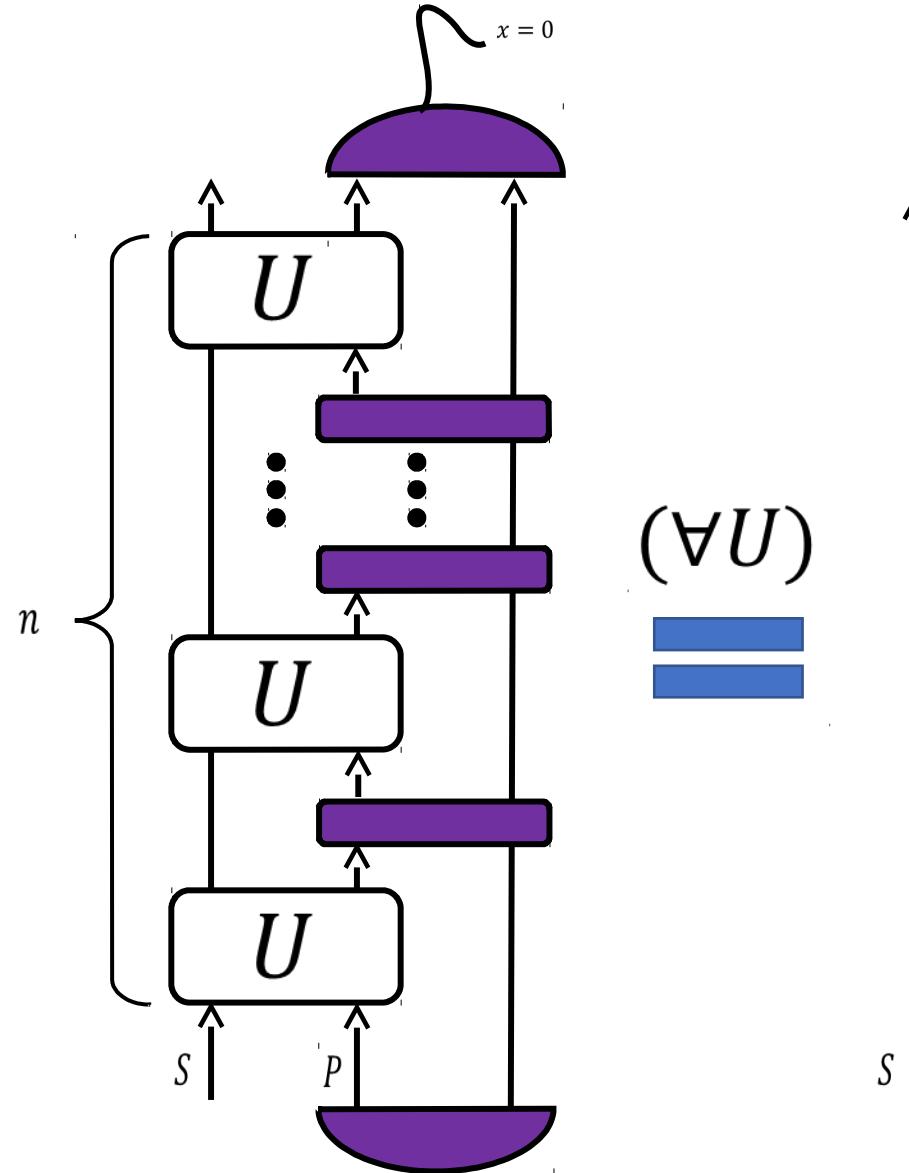
$$V = e^{-iH_0T}$$



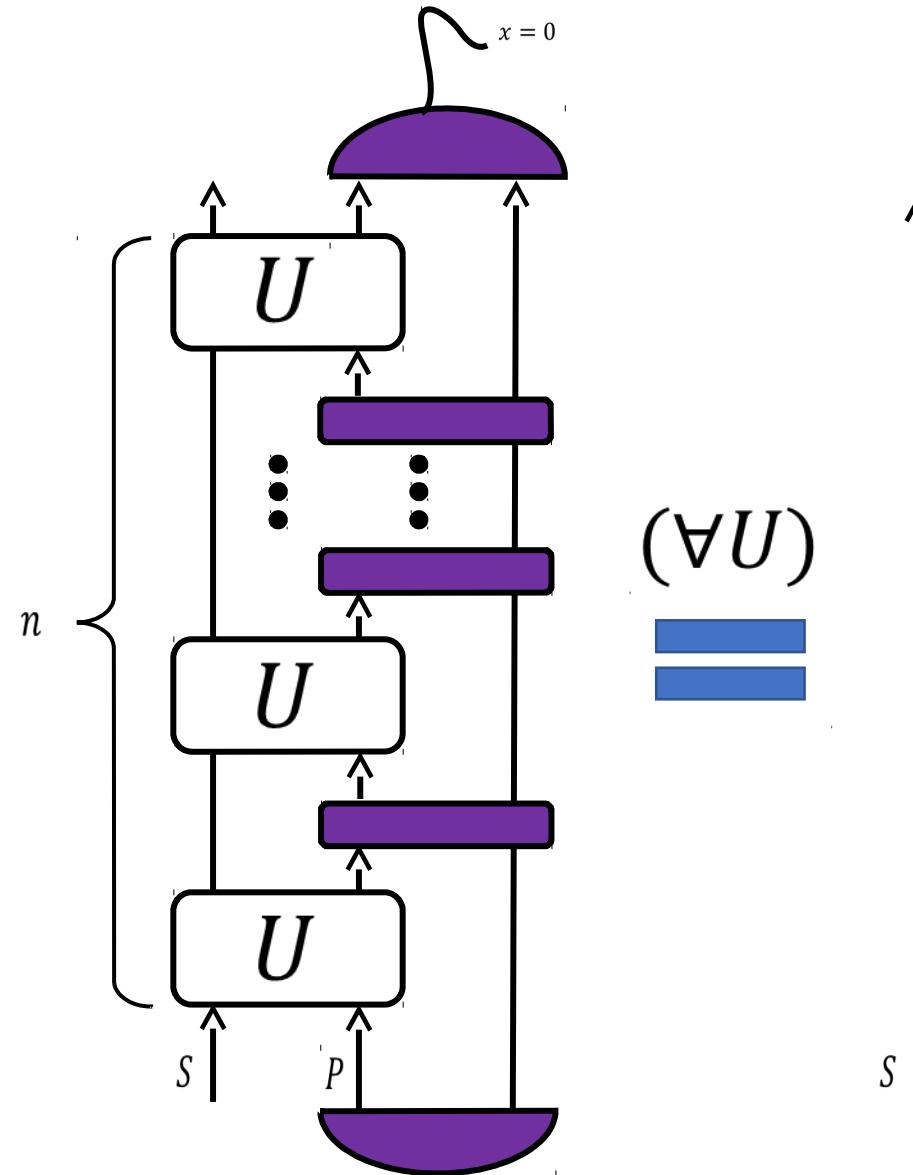




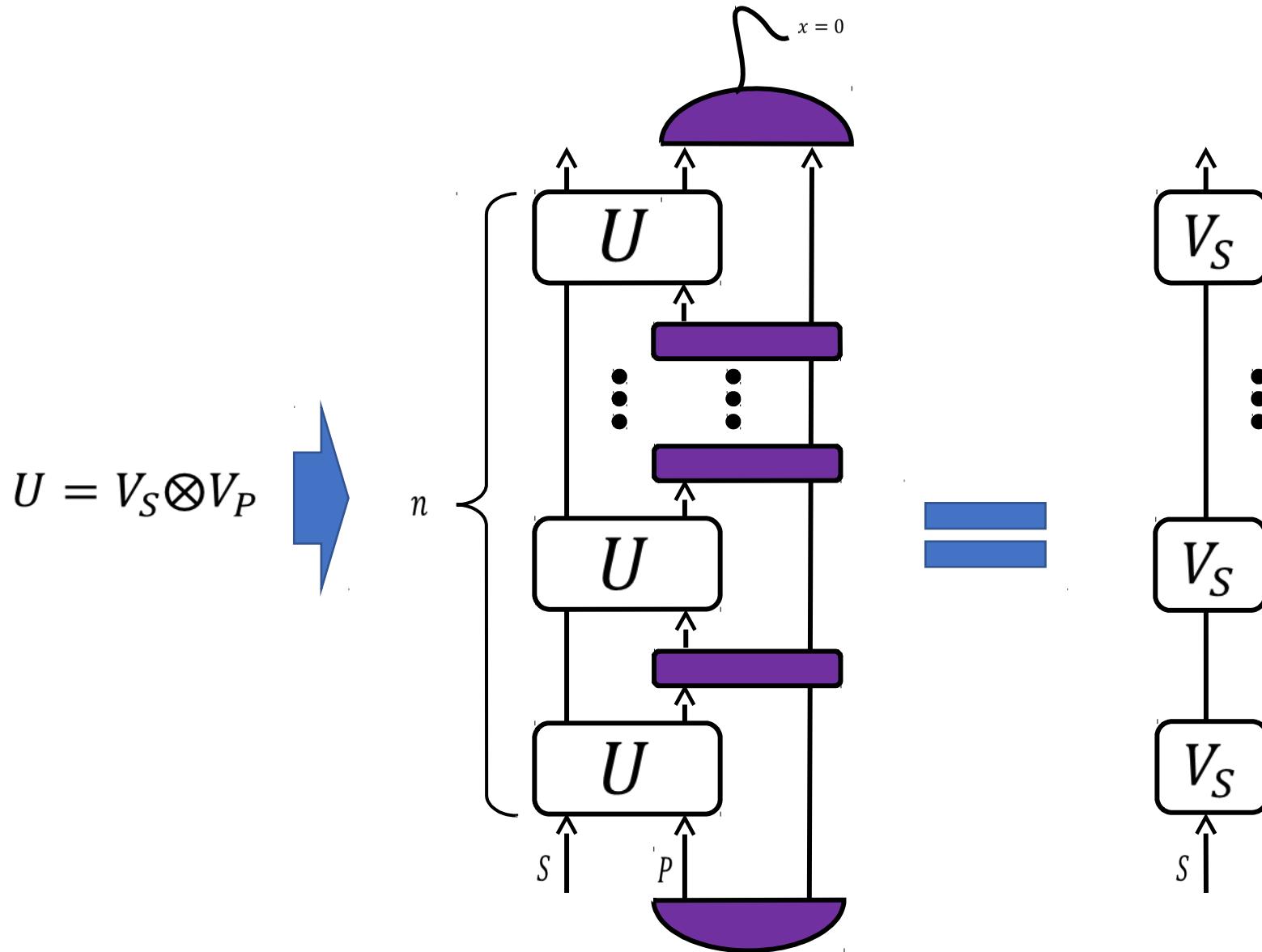
Quantum resetting
protocol



Quantum resetting
protocol



Desideratum: $P(x = 0) \neq 0$ for all U

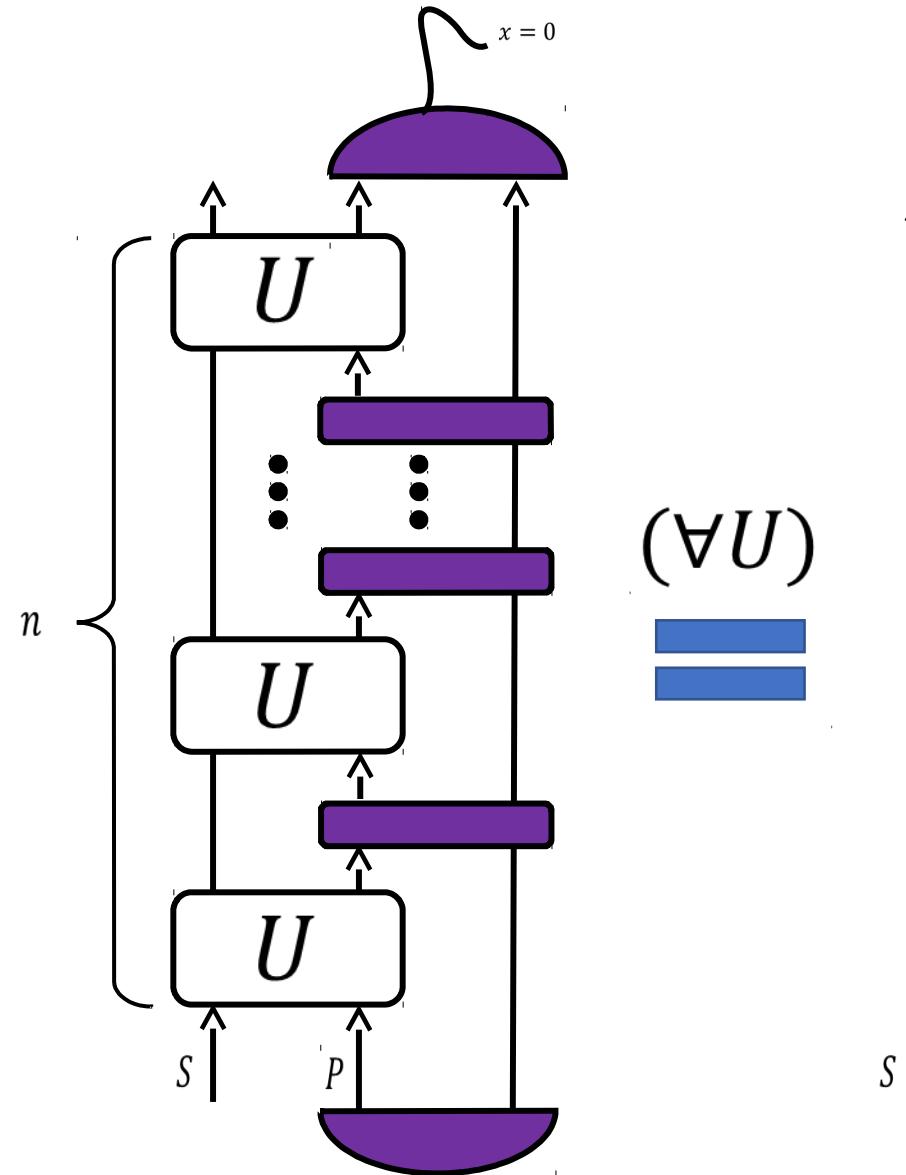


$$U = V_S \otimes V_P$$



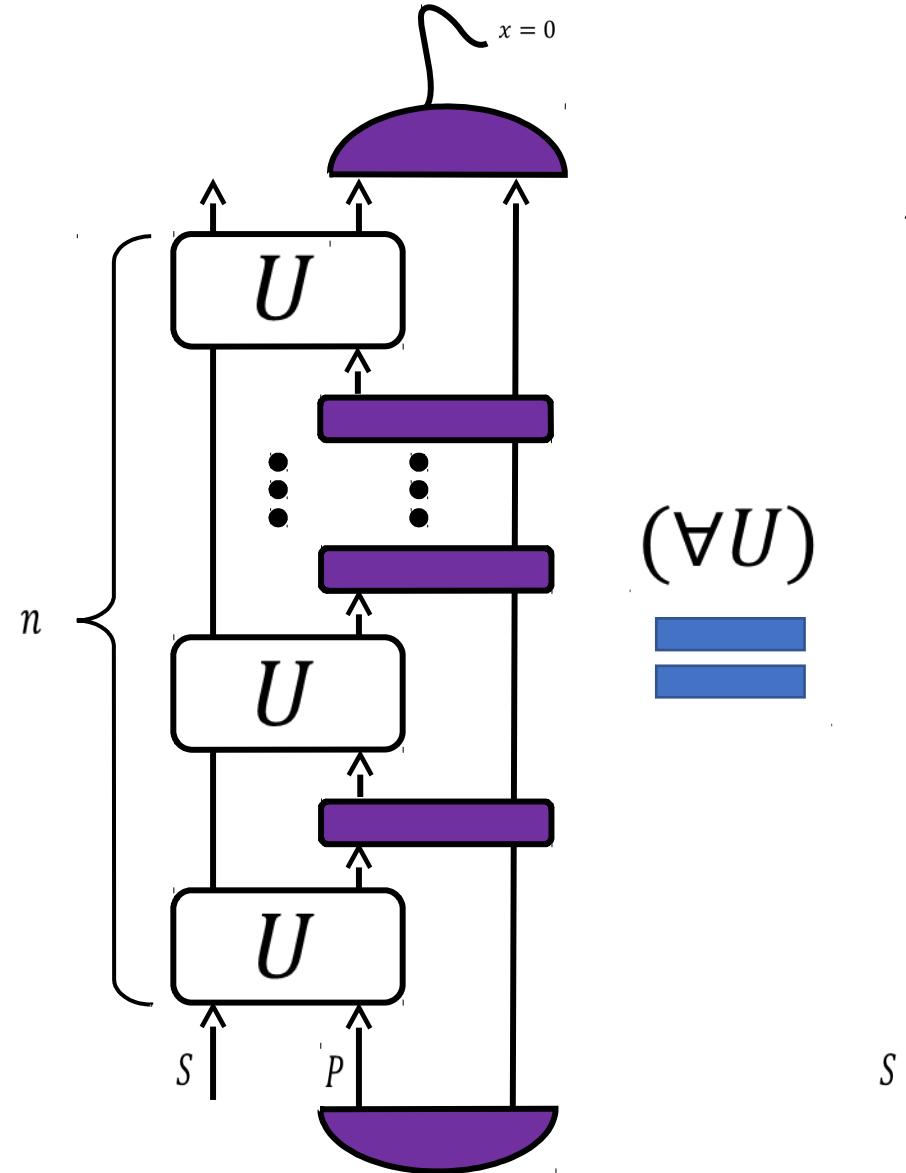
Resetting protocol will fail with probability 1

Quantum resetting
protocol



Desideratum: $P(x = 0 | U) = 1$ for all U

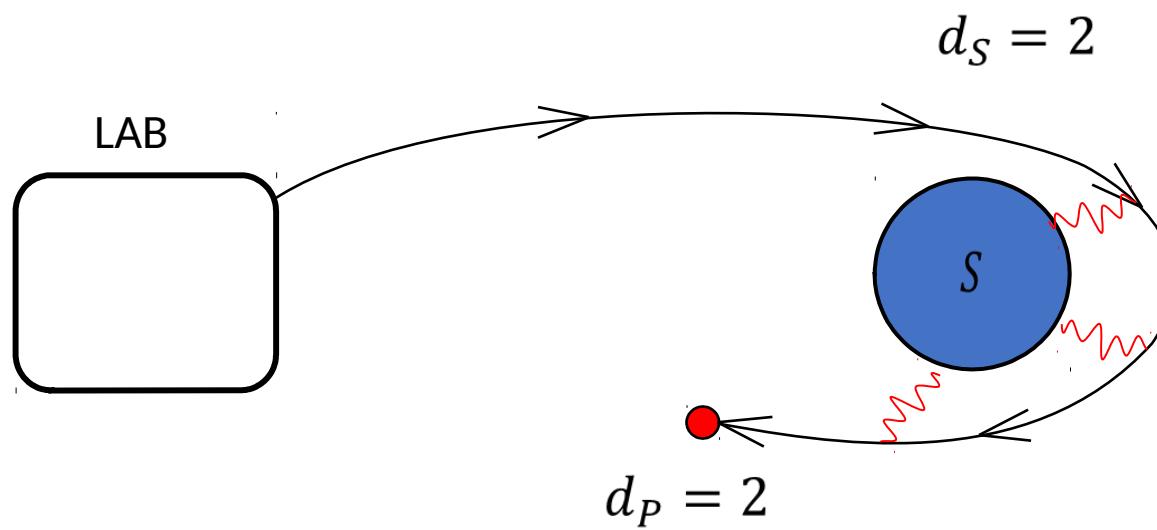
Quantum resetting
protocol



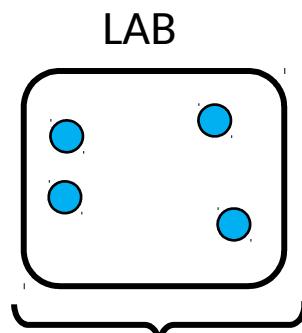
$P(x = 0|U) \neq 0$ except for a subset of unitaries of zero measure

Do quantum resetting protocols exist?

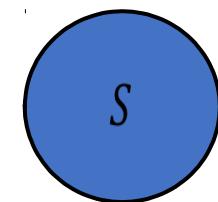
Scenario

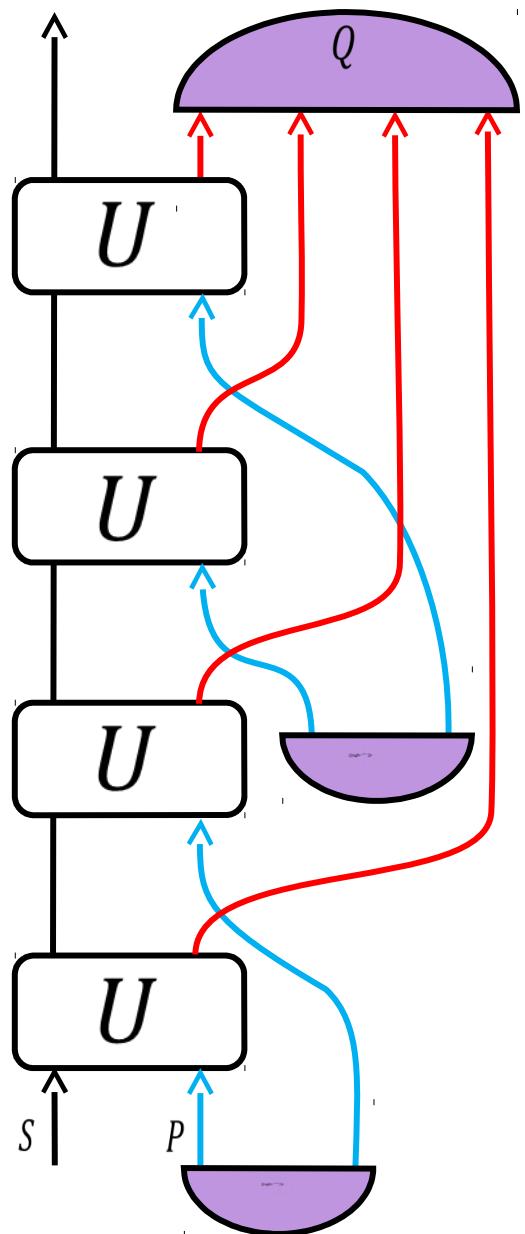


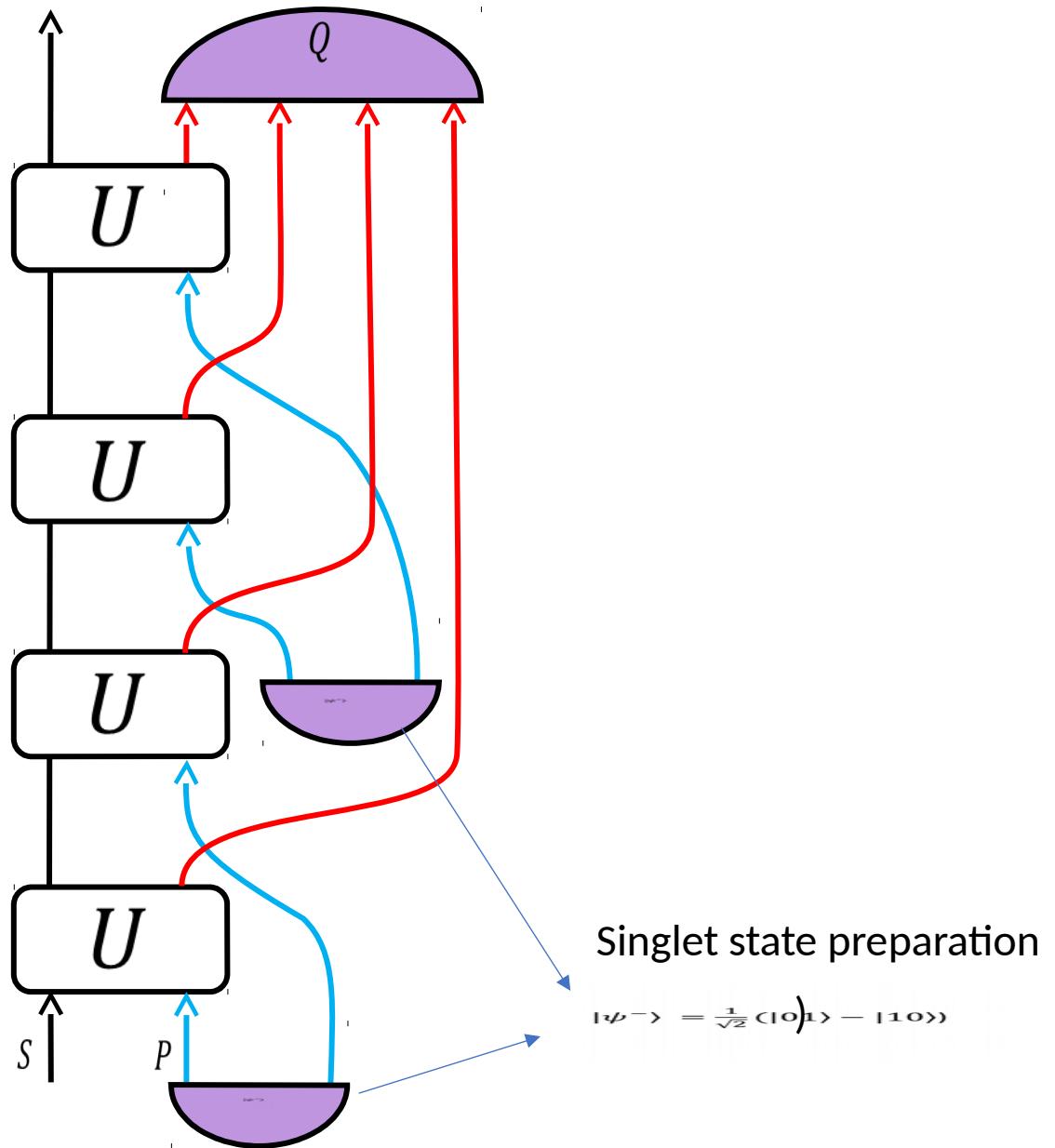
Scenario

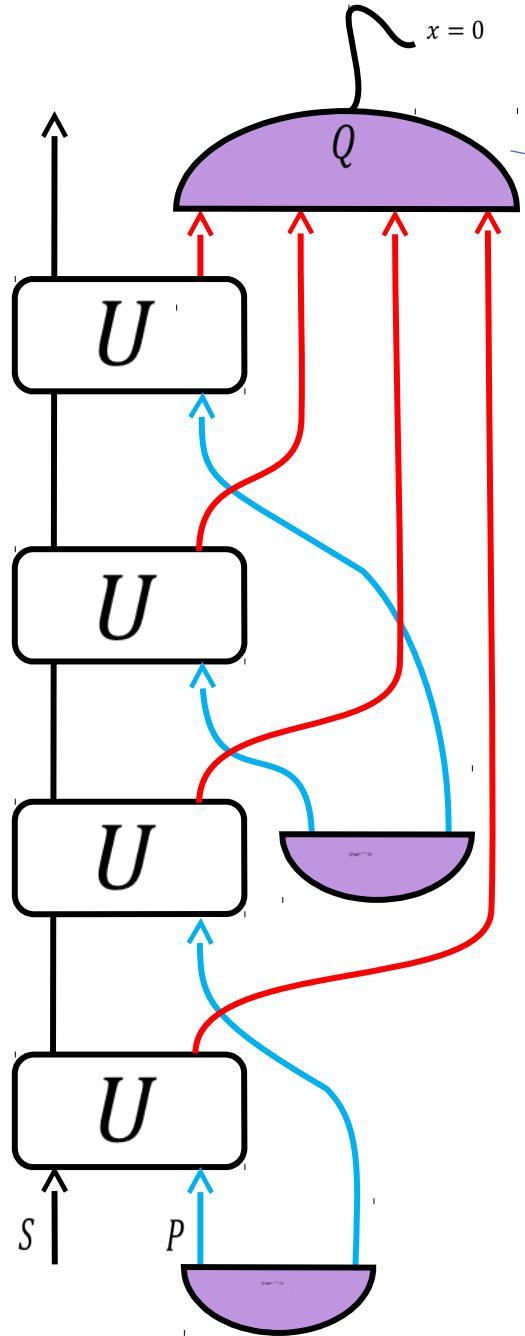


$$t = 4(T + \delta)$$









Projection onto the space
spanned by

$$|m_1\rangle = |0,0,0,0\rangle,$$

$$|m_2\rangle = \frac{1}{2}(|1,0,0,0\rangle + |0,1,0,0\rangle + |0,0,1,0\rangle + |0,0,0,1\rangle),$$

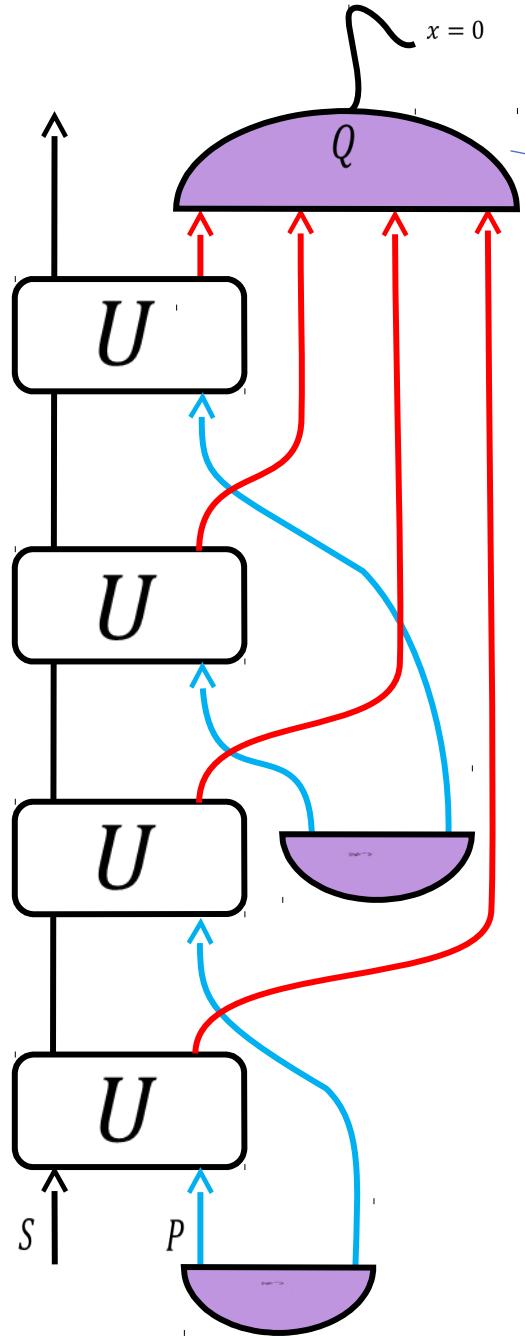
$$|m_3\rangle = \frac{1}{2}(|1,0,1,0\rangle + |0,1,0,1\rangle + |1,0,0,1\rangle + |0,1,1,0\rangle),$$

$$|m_4\rangle = \frac{1}{\sqrt{2}}(|0,0,1,1\rangle + |1,1,0,0\rangle),$$

$$|m_5\rangle = \frac{1}{2}(|1,1,1,0\rangle + |0,1,1,1\rangle + |1,0,1,1\rangle + |1,1,0,1\rangle),$$

$$|m_6\rangle = |1,1,1,1\rangle$$

Why does this work?



Projection onto the space
spanned by

$$|m_1\rangle = |0, 0, 0, 0\rangle,$$

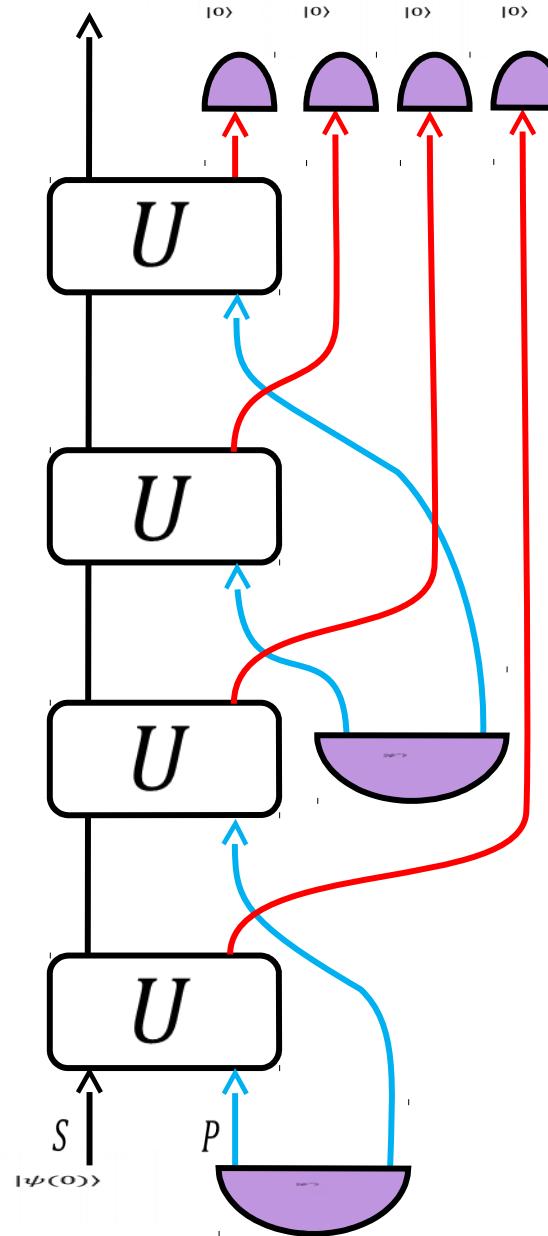
$$|m_2\rangle = \frac{1}{2}(|1, 0, 0, 0\rangle + |0, 1, 0, 0\rangle + |0, 0, 1, 0\rangle + |0, 0, 0, 1\rangle),$$

$$|m_3\rangle = \frac{1}{2}(|1, 0, 1, 0\rangle + |0, 1, 0, 1\rangle + |1, 0, 0, 1\rangle + |0, 1, 1, 0\rangle),$$

$$|m_4\rangle = \frac{1}{\sqrt{2}}(|0, 0, 1, 1\rangle + |1, 1, 0, 0\rangle),$$

$$|m_5\rangle = \frac{1}{2}(|1, 1, 1, 0\rangle + |0, 1, 1, 1\rangle + |1, 0, 1, 1\rangle + |1, 1, 0, 1\rangle),$$

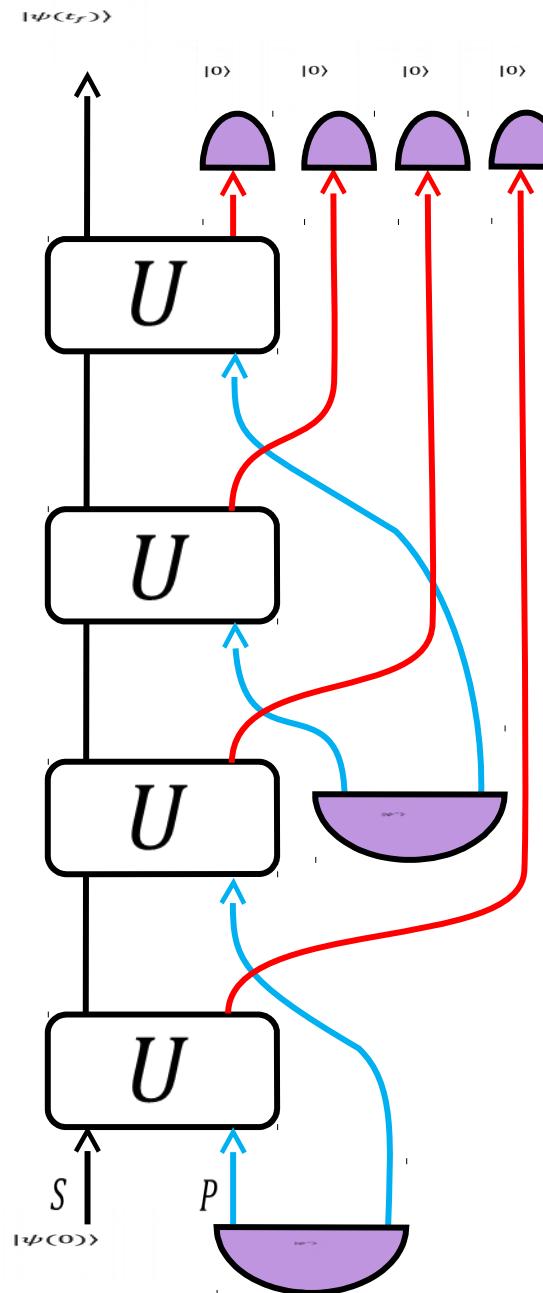
$$|m_6\rangle = |1, 1, 1, 1\rangle$$

$|\psi(t_f)\rangle$ 

$$U_{ij} = (\mathbb{I}_S \otimes \langle i |_P) U (\mathbb{I}_S \otimes |j\rangle_P)$$

2x2 complex matrices

$$|\psi(t_f)\rangle = \frac{1}{2} [U_{0,0}, U_{0,1}]^2 |\psi(0)\rangle$$





$A, B, 2 \times 2$ matrices

A, B , 2×2 matrices

$$[A, B] = \sum_{i=0,1,2,3} c_i \sigma_i$$

σ_i , Pauli matrices

A, B , 2×2 matrices

$$[A, B] = \sum_{i=0,1,2,3} c_i \sigma_i \quad \text{Tr}([A, B]) = 0 \rightarrow c_0 = 0$$

σ_i , Pauli matrices

A, B , 2×2 matrices

$$[A, B] = \sum_{i=1,2,3} c_i \sigma_i$$

σ_i , Pauli matrices

A, B , 2×2 matrices

$$[A, B]^2 = \left(\sum_{i=1,2,3} c_i \sigma_i \right)^2 = \left(\sum_{i=1,2,3} c_i^2 \right) \mathbb{I}$$

σ_i , Pauli matrices

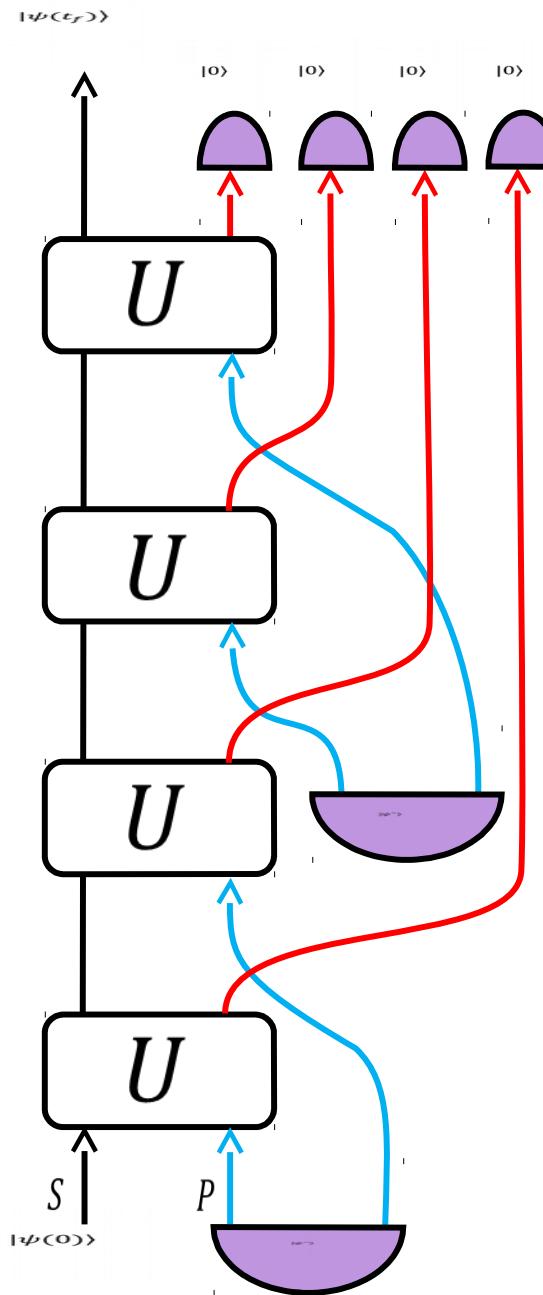
Central polynomial for dimension 2

$A, B, 2 \times 2$ matrices

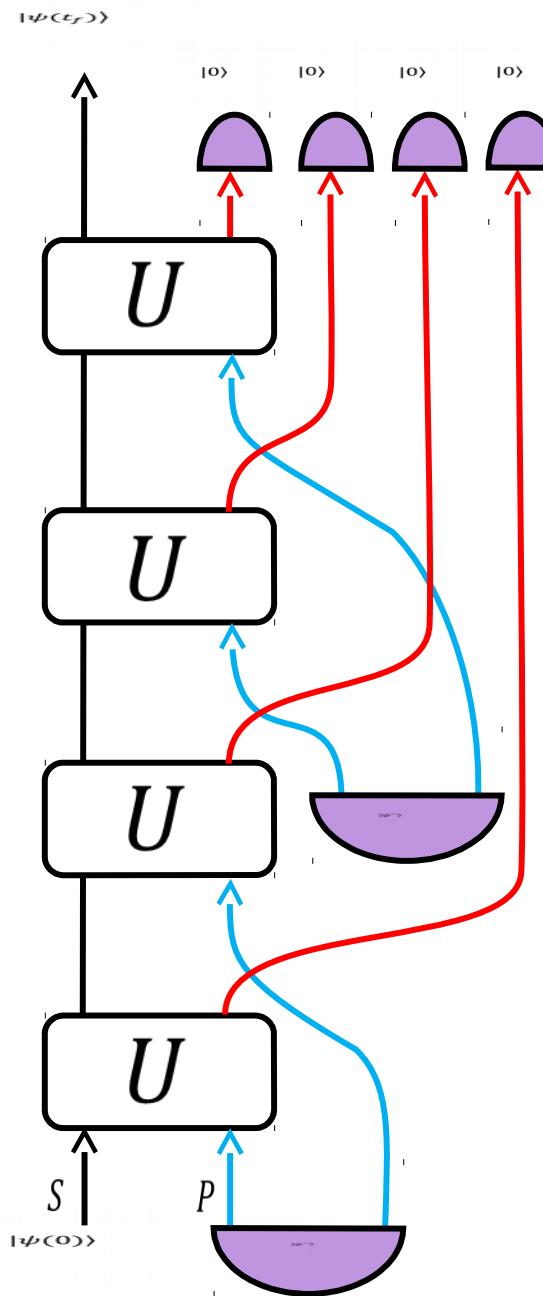
$$[A, B]^2 = \left(\sum_{i=1,2,3} c_i \sigma_i \right)^2 = \left(\sum_{i=1,2,3} c_i^2 \right) \mathbb{I}$$

σ_i , Pauli matrices

$$|\psi(t_f)\rangle = \frac{1}{2} [U_{0,0}, U_{0,1}]^2 |\psi(0)\rangle$$

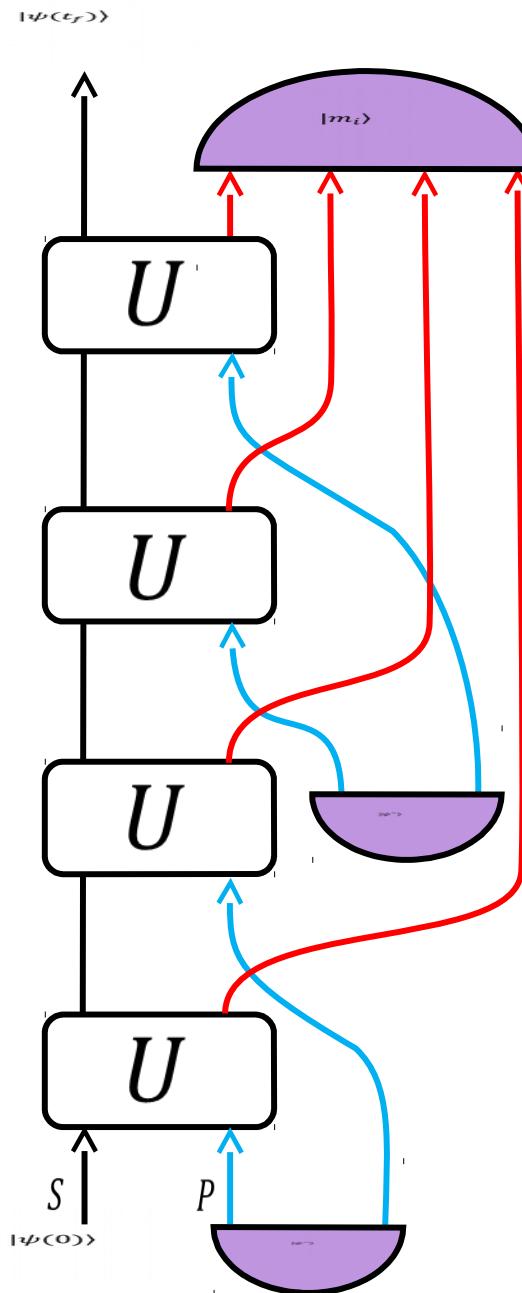


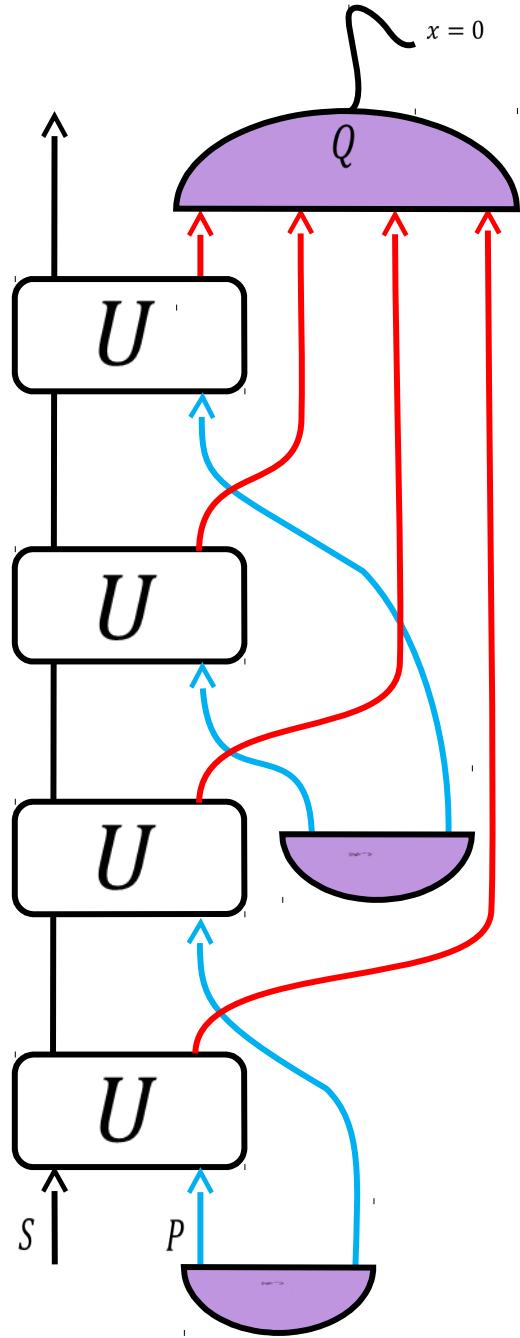
$$|\psi(t_f)\rangle = \frac{1}{2} [U_{0,0}, U_{0,1}]^2 |\psi(0)\rangle \propto |\psi(0)\rangle$$



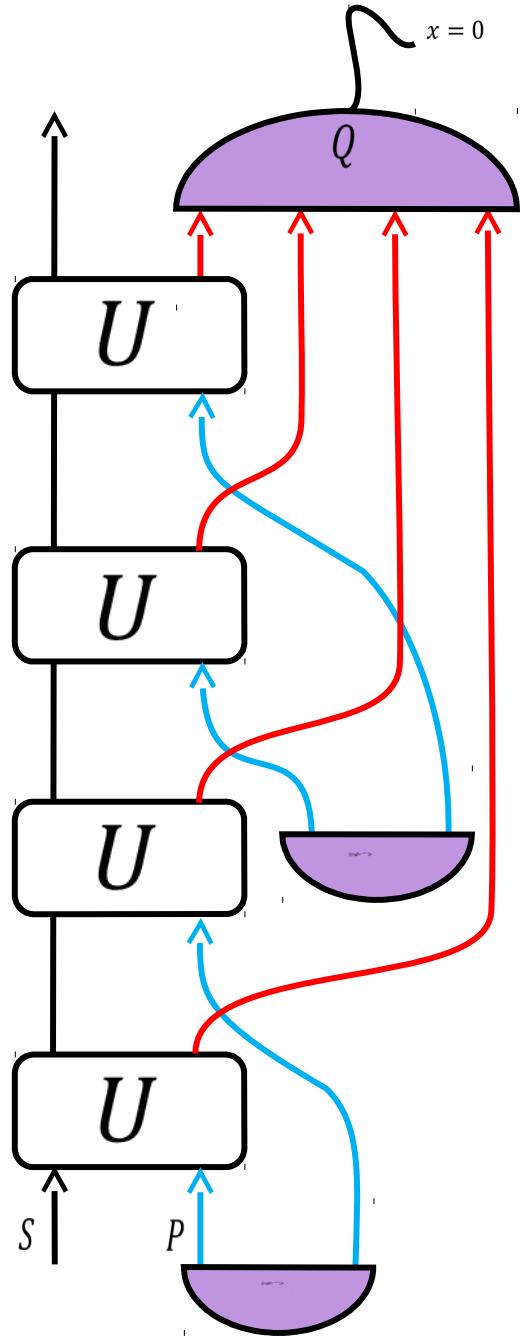
Similarly,

$$\begin{aligned}
 |m_1\rangle &= |0,0,0,0\rangle, \\
 |m_2\rangle &= \frac{1}{2}(|1,0,0,0\rangle + |0,1,0,0\rangle + |0,0,1,0\rangle + |0,0,0,1\rangle), \\
 |m_3\rangle &= \frac{1}{2}(|1,0,1,0\rangle + |0,1,0,1\rangle + |1,0,0,1\rangle + |0,1,1,0\rangle), \\
 |m_4\rangle &= \frac{1}{\sqrt{2}}(|0,0,1,1\rangle + |1,1,0,0\rangle), \\
 |m_5\rangle &= \frac{1}{2}(|1,1,1,0\rangle + |0,1,1,1\rangle + |1,0,1,1\rangle + |1,1,0,1\rangle), \\
 |m_6\rangle &= |1,1,1,1\rangle
 \end{aligned}$$



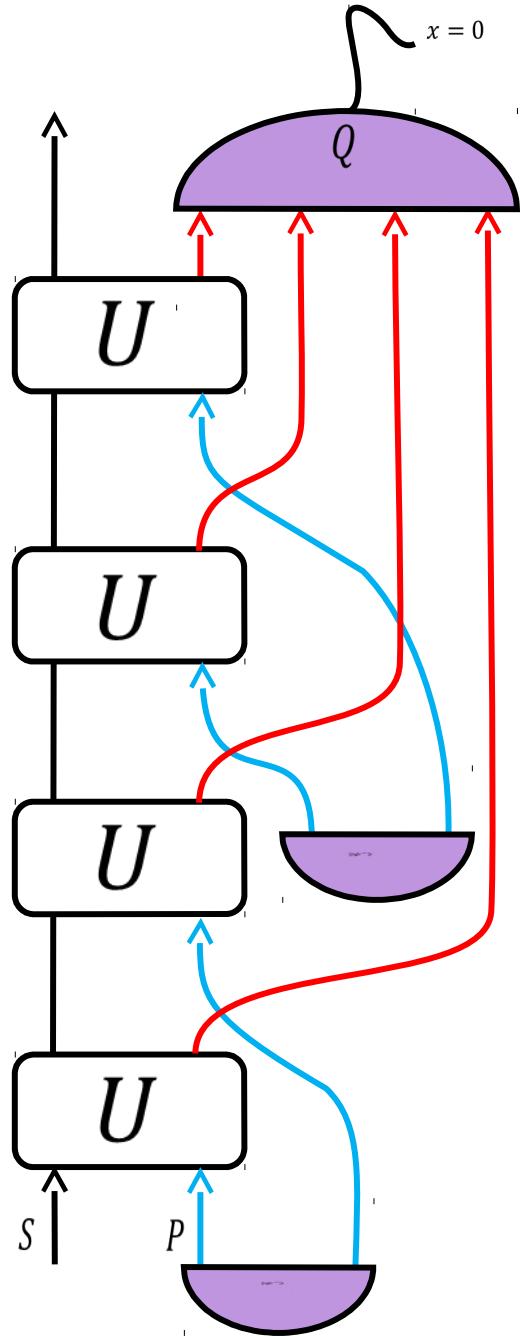


$$P(x = 0|U) \in [0, 1]$$



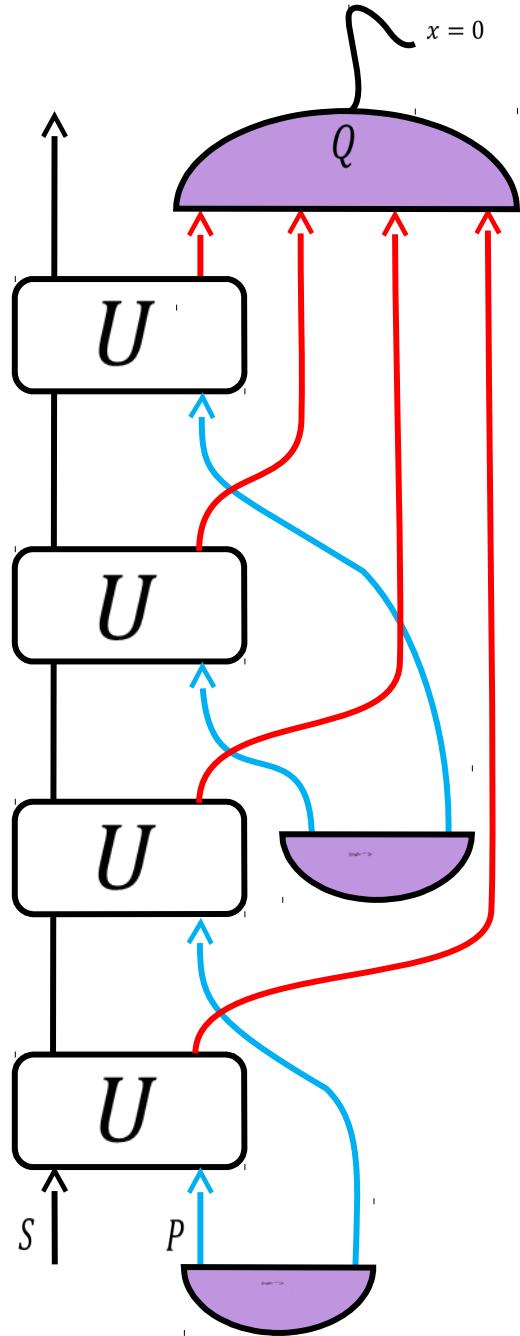
$$P(x = 0|U) \in [0, 1]$$

E.g.: $U_{E\bar{g}..}^{\bar{e}} V_S \otimes V_P$



$$P(x = 0|U) \in [0, 1]$$

E.g.: $U = \frac{\sigma_x \otimes \sigma_z + i\sigma_y \otimes \sigma_x}{\sqrt{2}}$



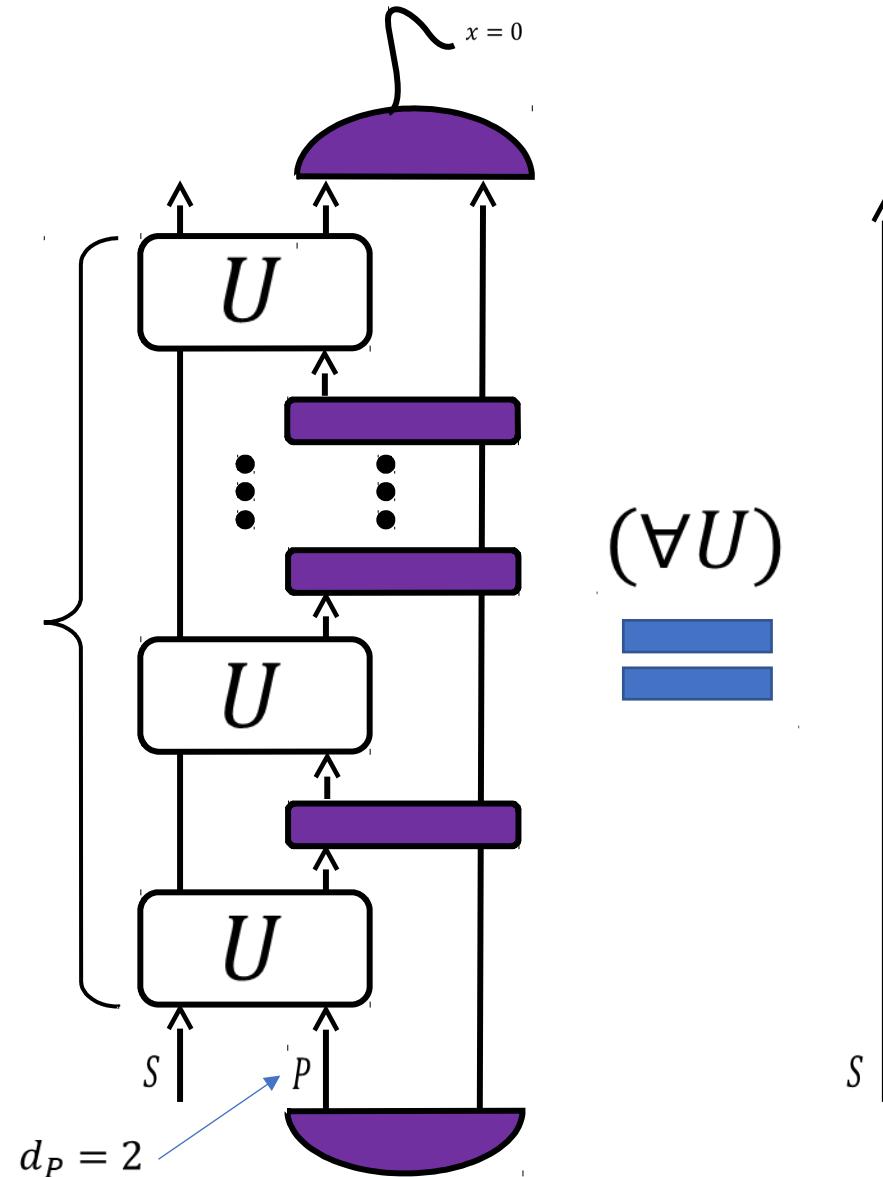
Average probability of success for completely unknown U

$$\int dU P(x = 0|U) \approx 0.2170$$

Generalization

For all d_S

$$\nabla = O(d_S^3) \\ [O(d_S^2 \log d_S)]$$



$P(x = 0|U) \neq 0$ except for a subset of unitaries of zero measure

Characterization of all quantum resetting protocols (practical for n=4)

$$\max \text{tr}(M_0^T X(\rho))$$

$$\text{s.t. } \text{supp}(M_0^T) \in \mathcal{H}^c, M_0, M_1 \geq 0$$

$$M_0 + M_1 = \mathbb{I}_{A_n^{out}} \otimes \Gamma^{(n)},$$

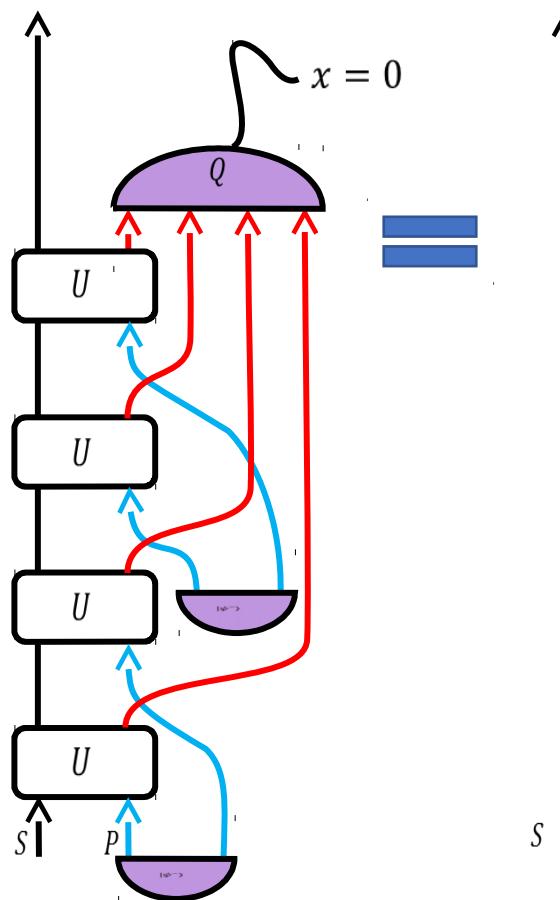
$$\text{tr}_{I_k}(\Gamma^{(k)}) = \mathbb{I}_{O_{k-1}} \otimes \Gamma^{(k-1)},$$

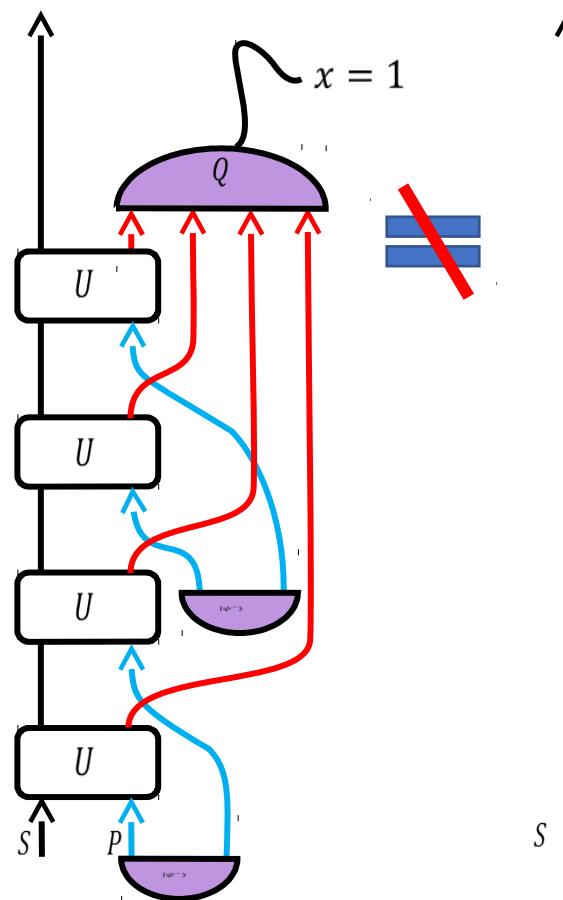
Heuristics to identify optimal strategies for high numbers of probes

$$\mathcal{W}_8, \tilde{\mathcal{W}}_8, \quad n = 8, d_S = 2$$

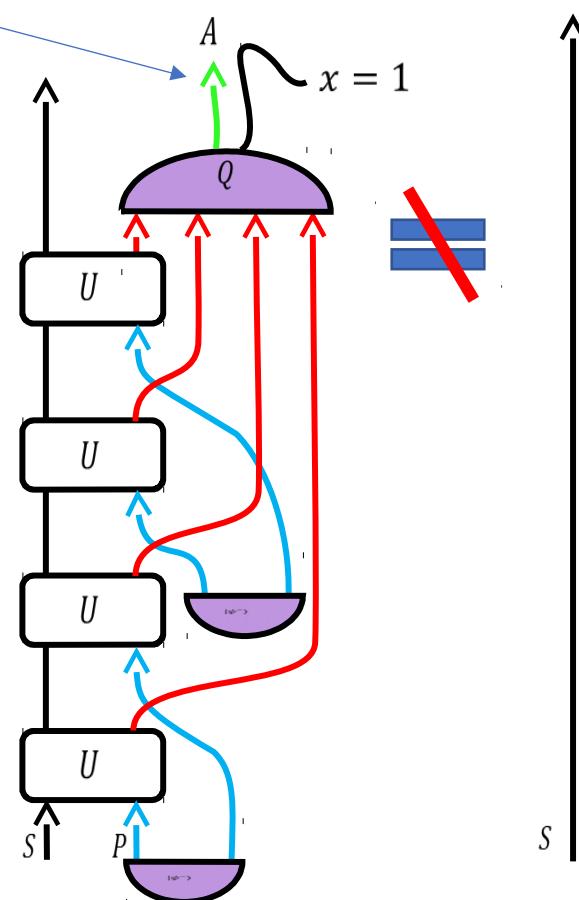
$$\mathcal{W}_9 \quad n = 9, d_S = 3$$

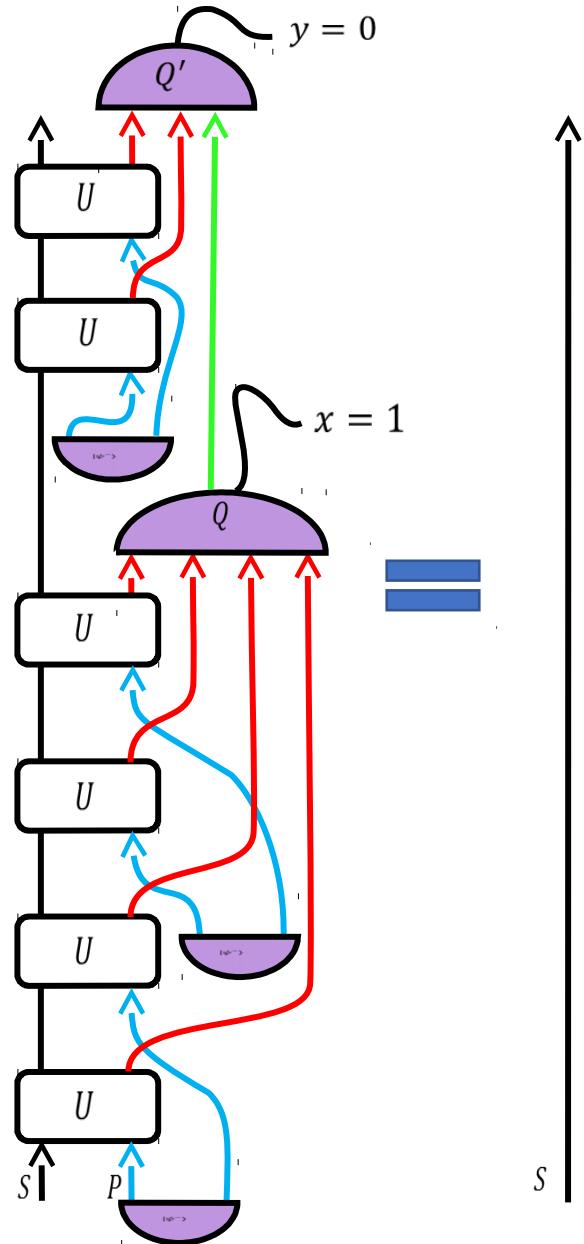
What if the protocol fails?





Suppose that the
last measurement
is a non-
demolition one

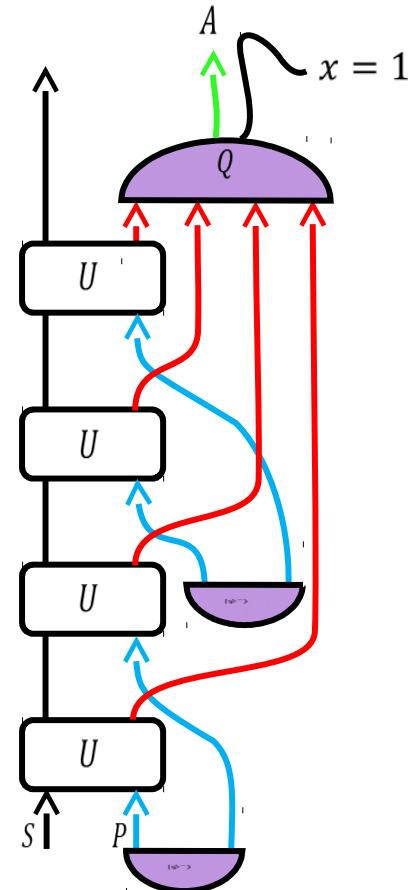




$$|\omega\rangle_{SA} = \sum_i f_i(U) |\psi\rangle_S |i\rangle_A$$

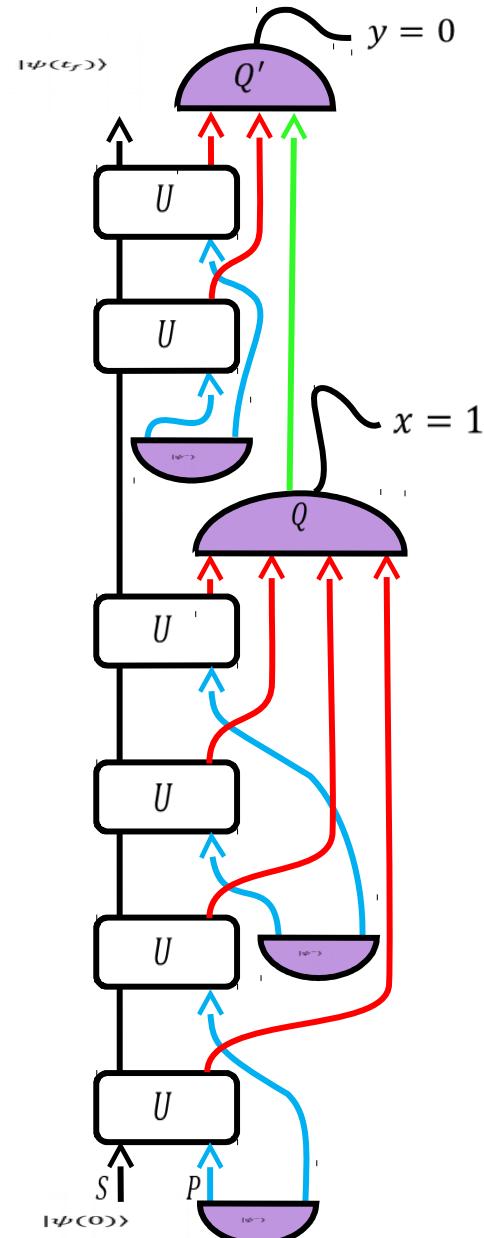
f_i non-central matrix polynomials
of degree 4 on the variables

$$U_{ij} = (\mathbb{I}_S \otimes \langle i |_P) U (\mathbb{I}_S \otimes |j \rangle_P)$$

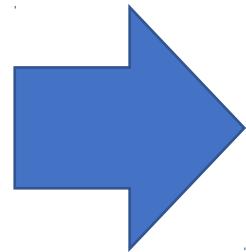


$$|\psi(t_f)\rangle_S = \sum_i g_i(U) f_i(U) |\psi(0)\rangle_S$$

matrix polynomials of degree 2 such
that $\sum_i g_i(U) f_i(U)$ is a central
polynomial



Undoing six possible failures

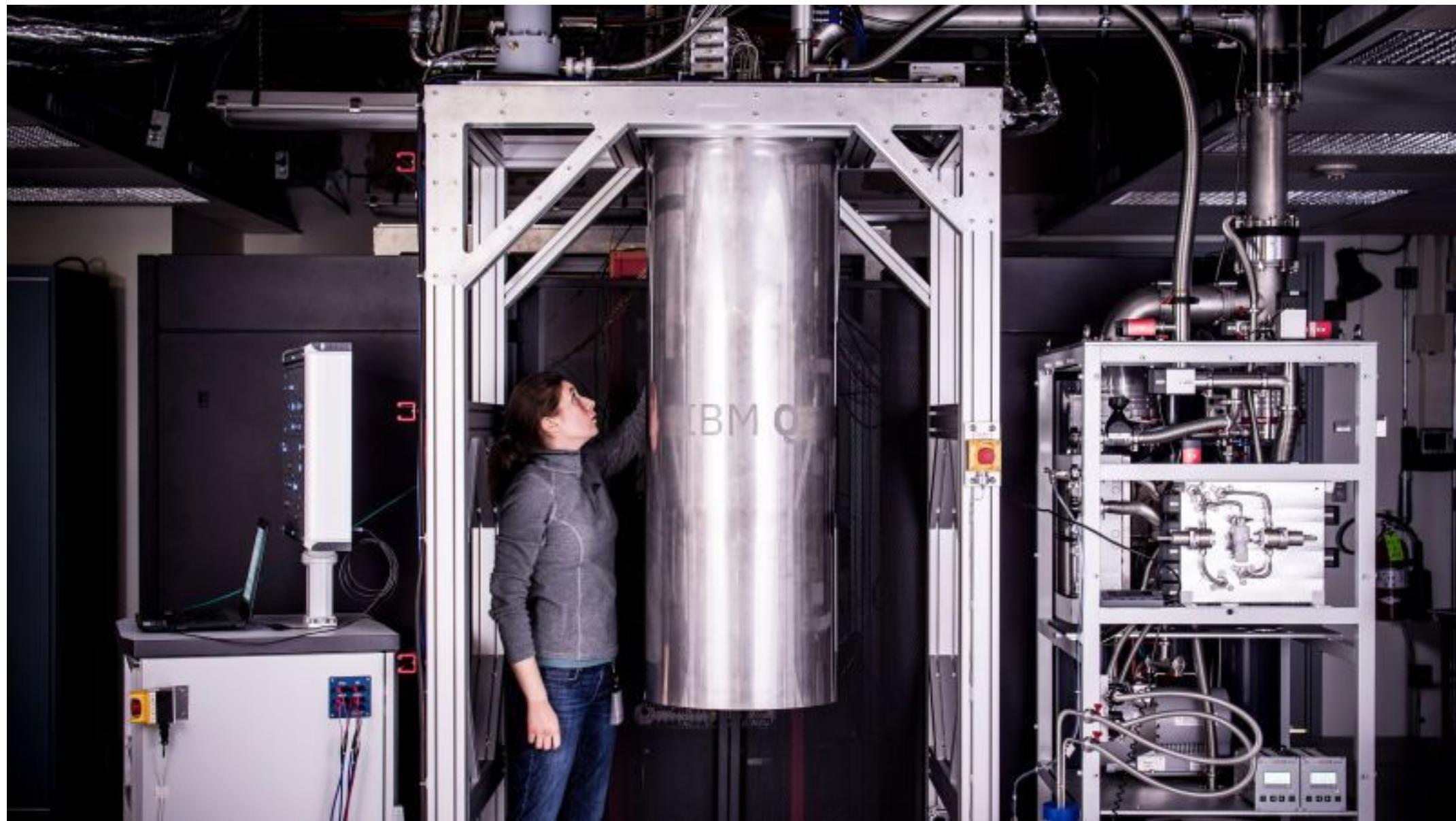


Average probability of success for completely unknown U

$$\int dU P(x = 0|U) \approx 0.6585$$

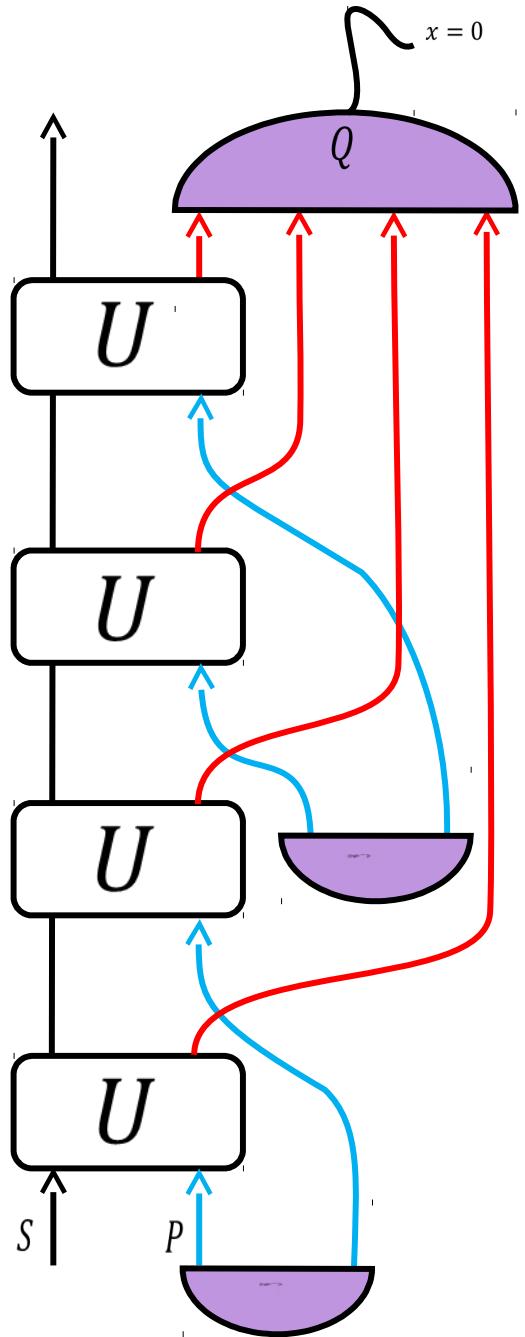


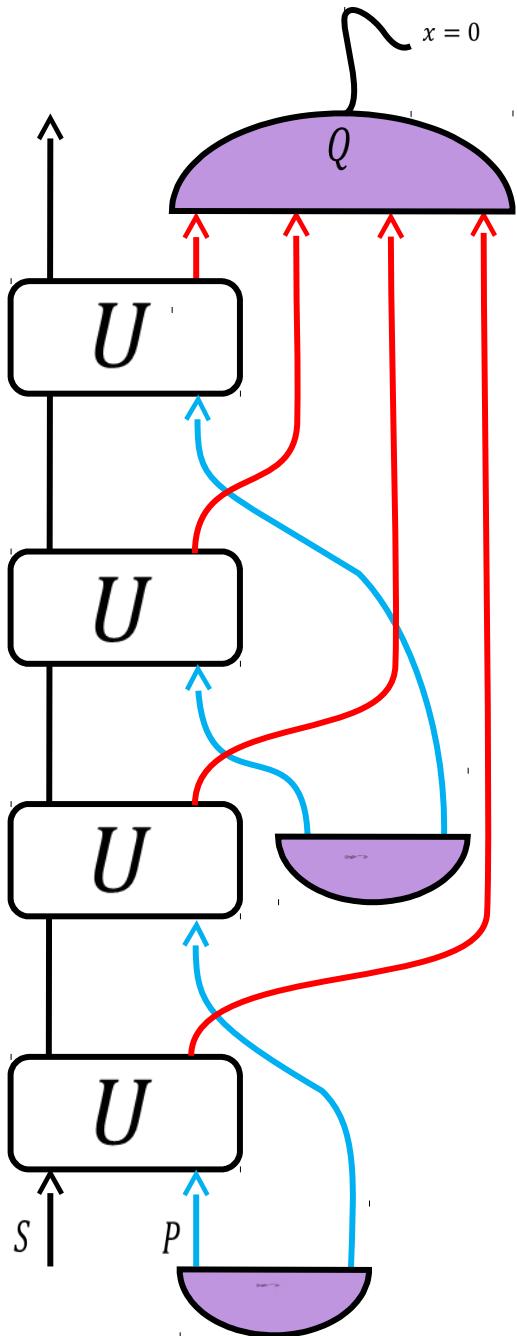
Experimental implementation?



IBM Q Experience

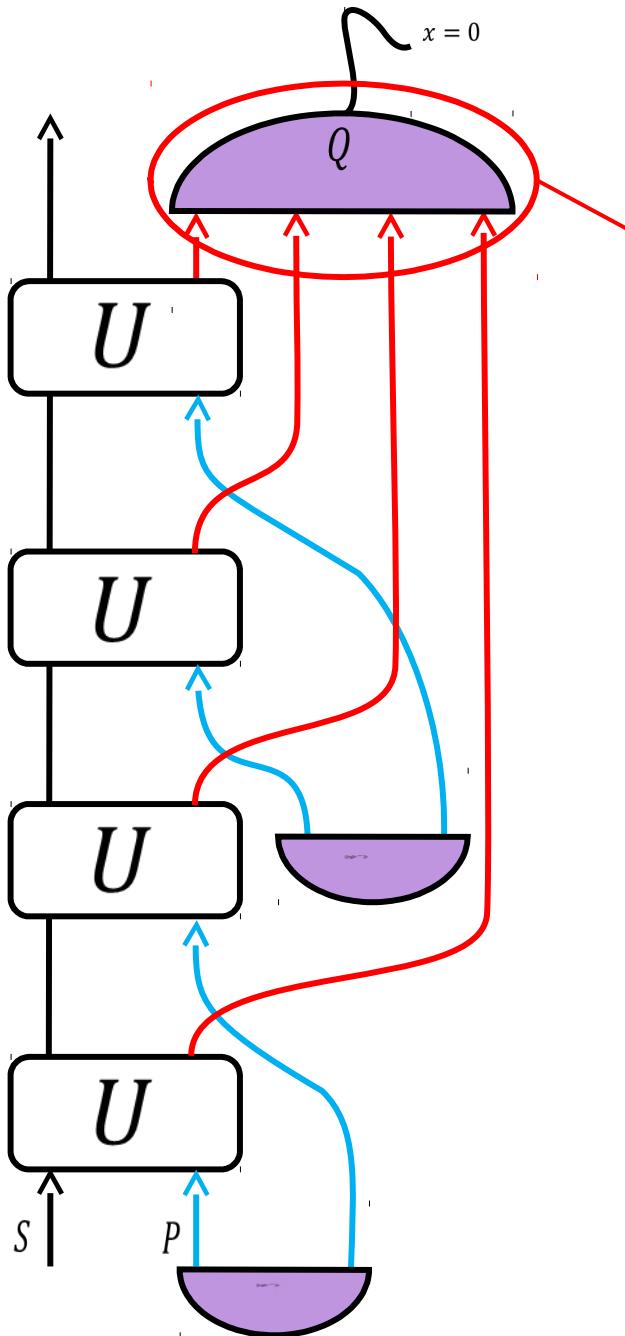
Experimental implementation: Prototype II



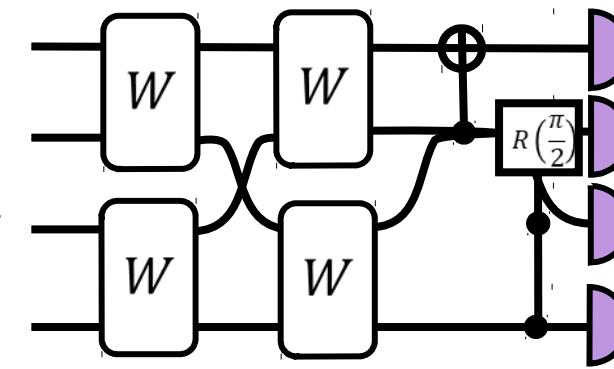
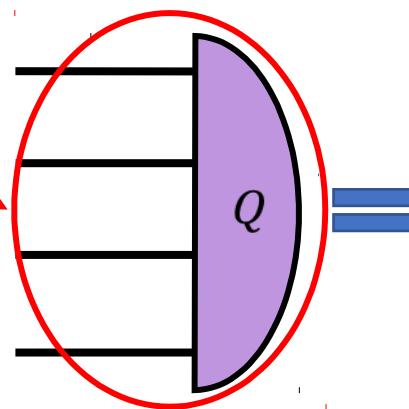


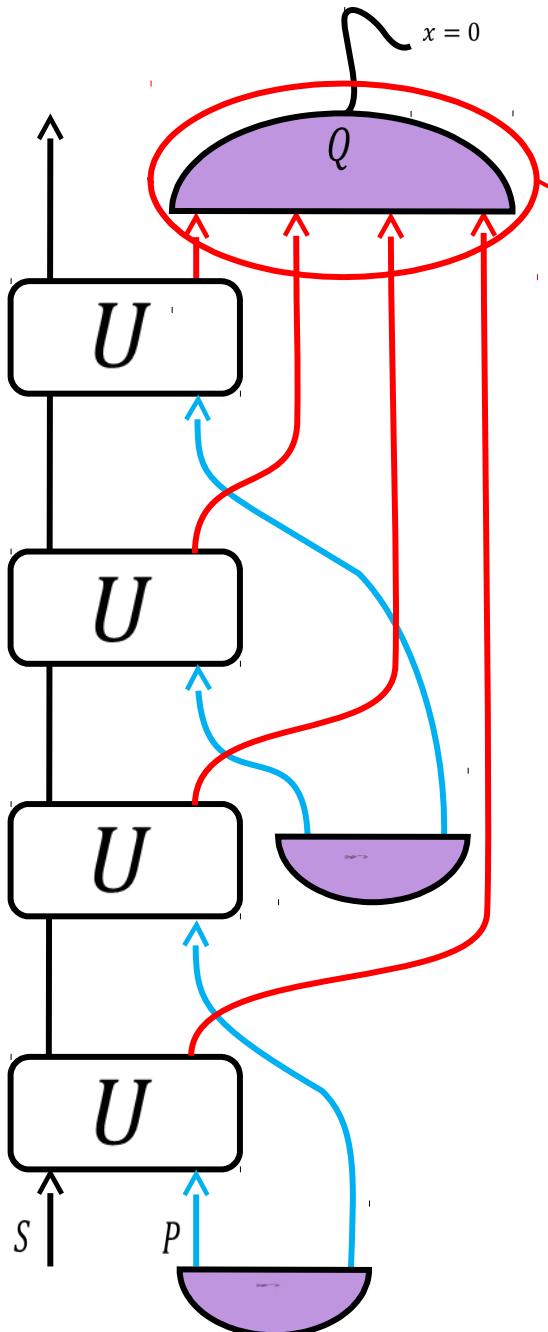
Experimental implementation: Prototype II

Implementation with single-qubit gates and CNOTS?

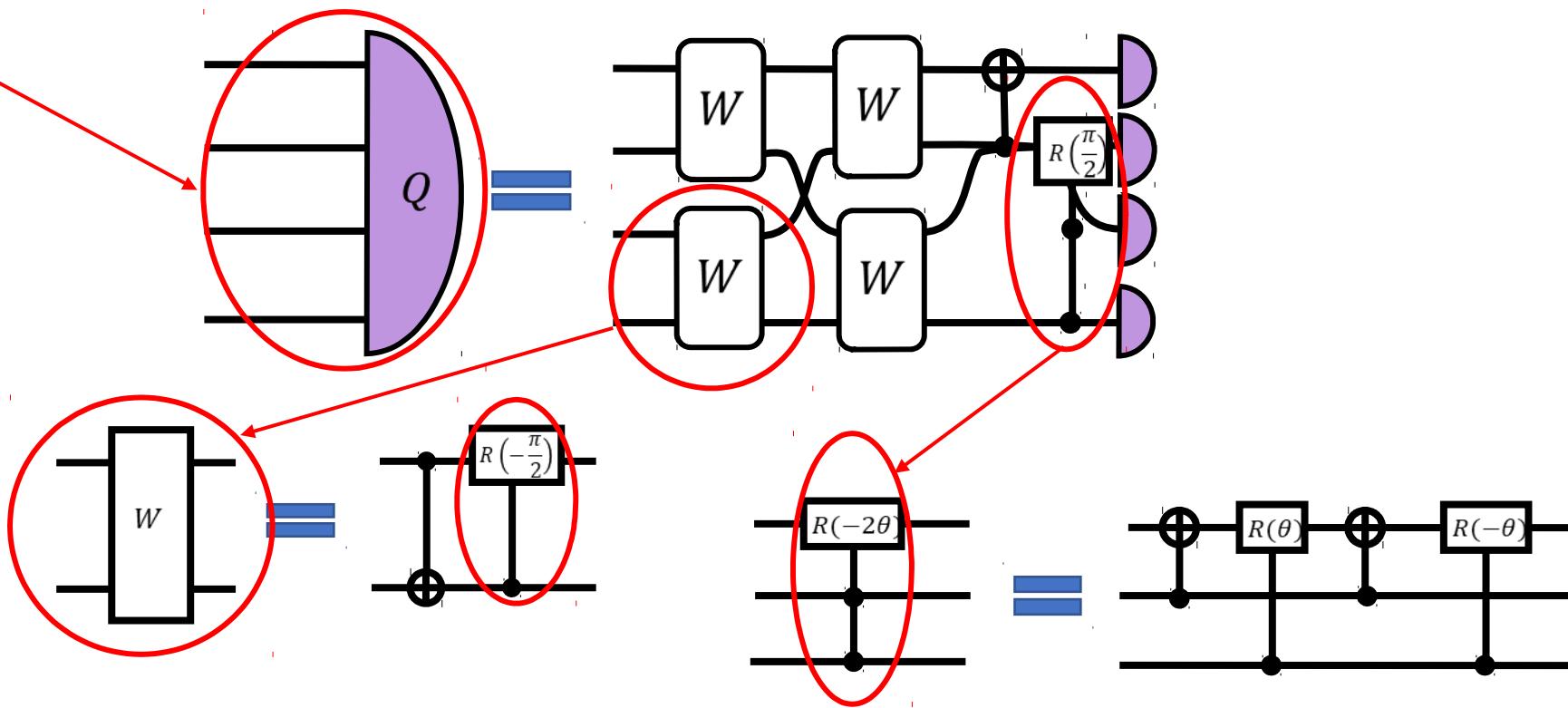


Experimental implementation: Prototype II

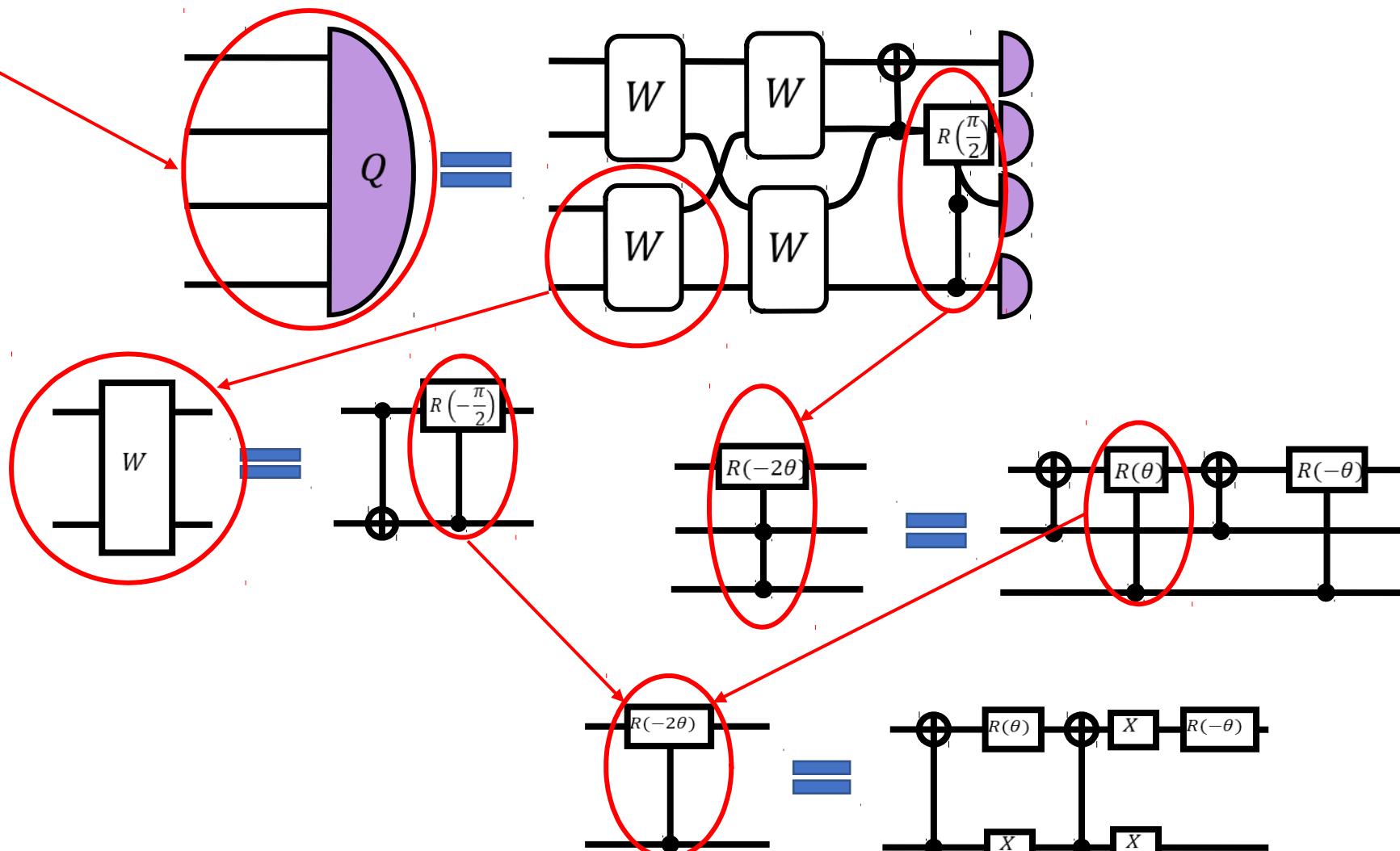
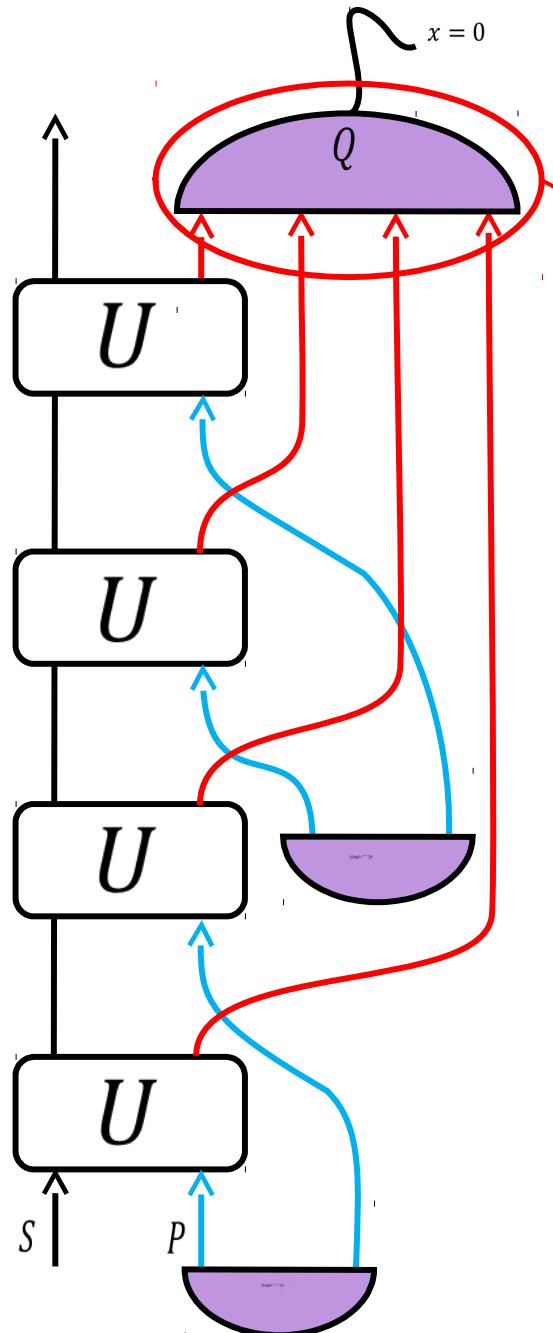




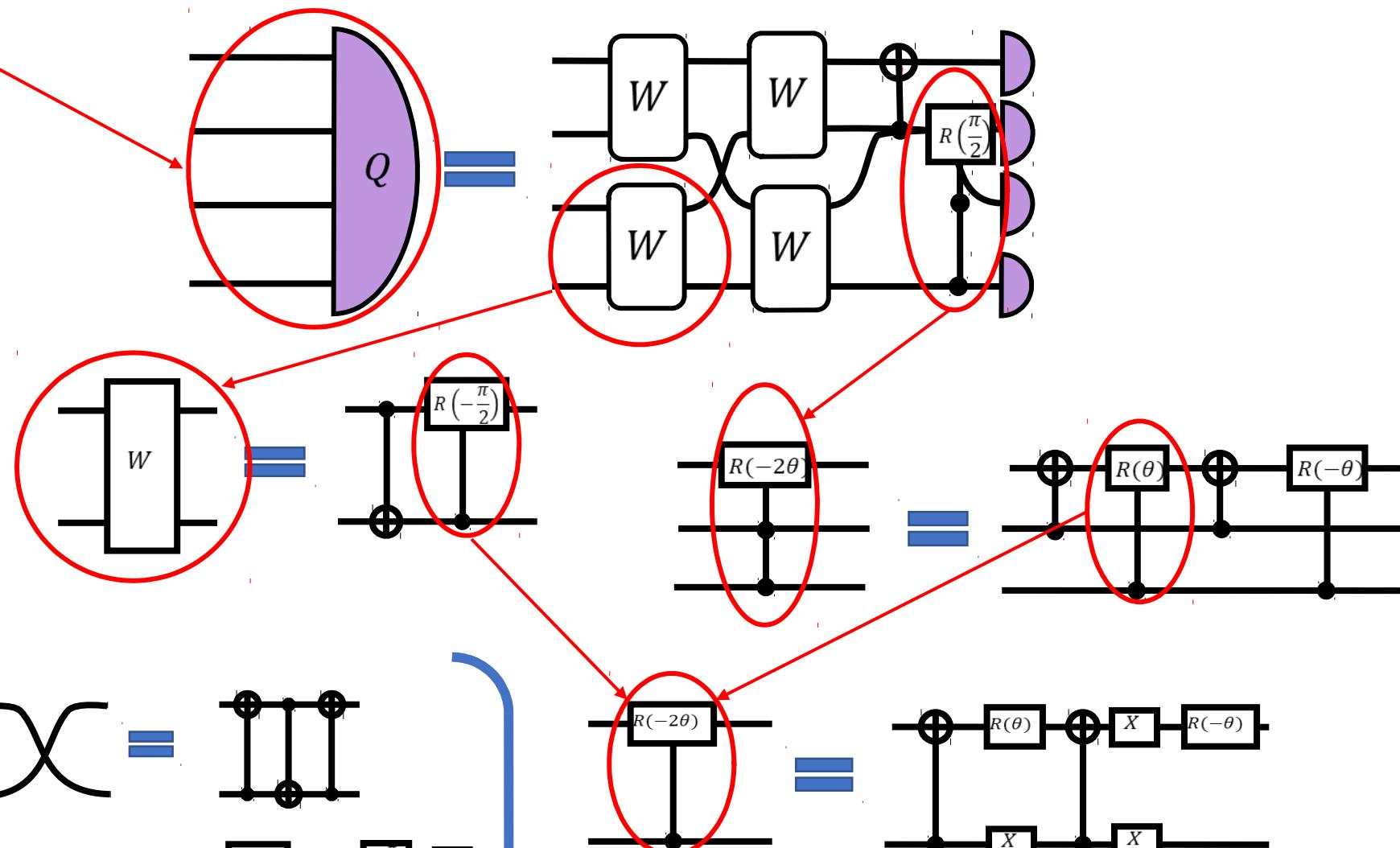
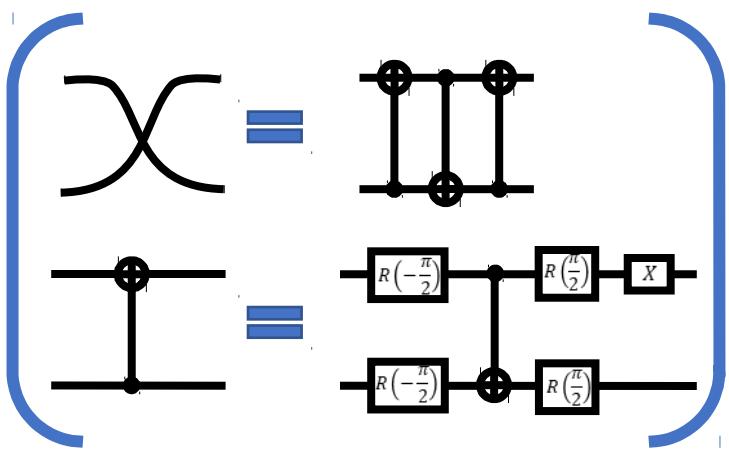
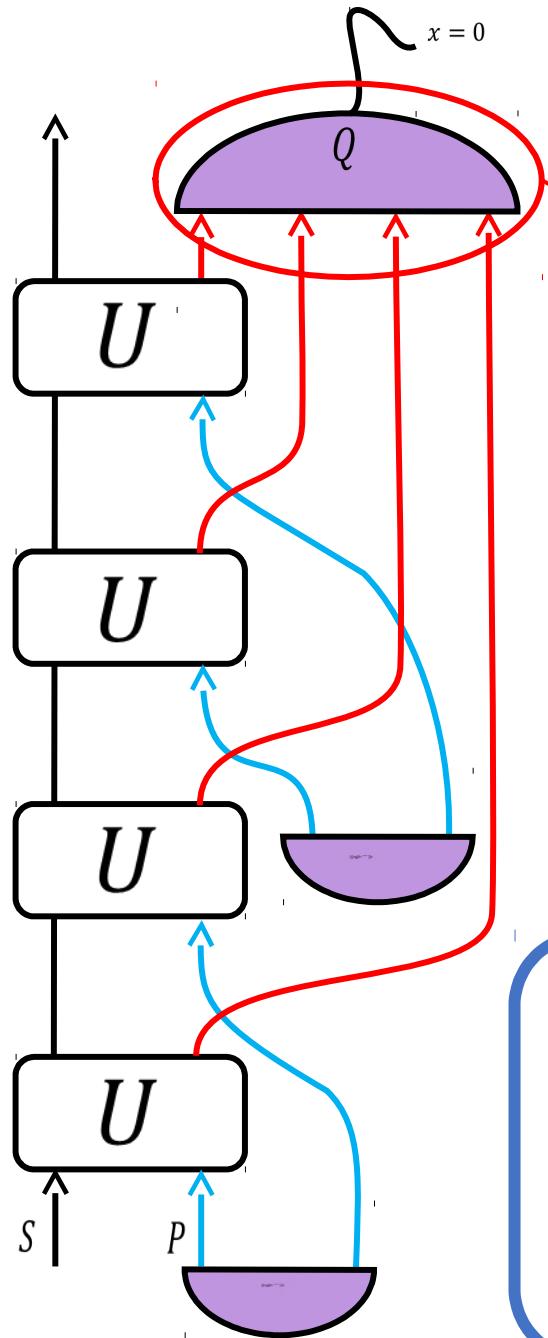
Experimental implementation: Prototype II

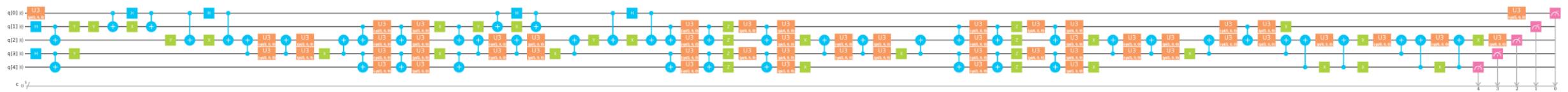
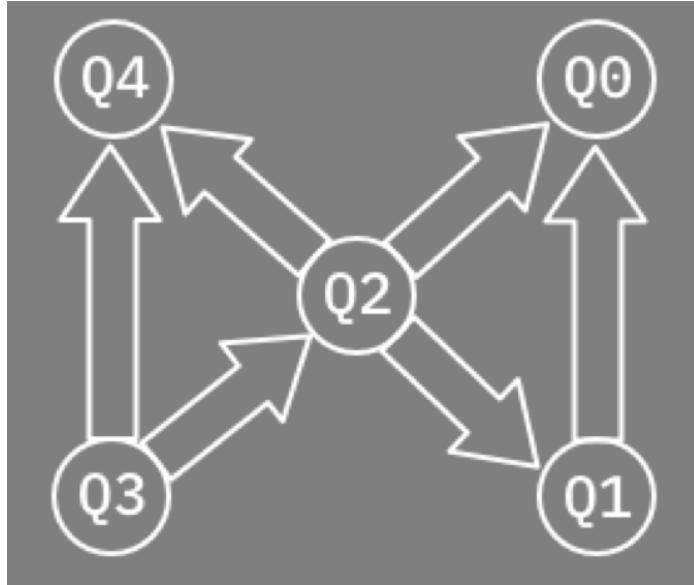


Experimental implementation: Prototype II



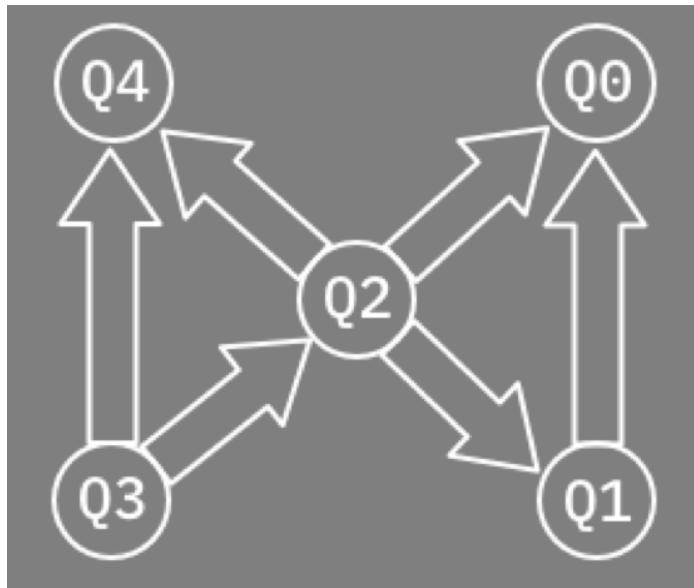
Experimental implementation: Prototype II



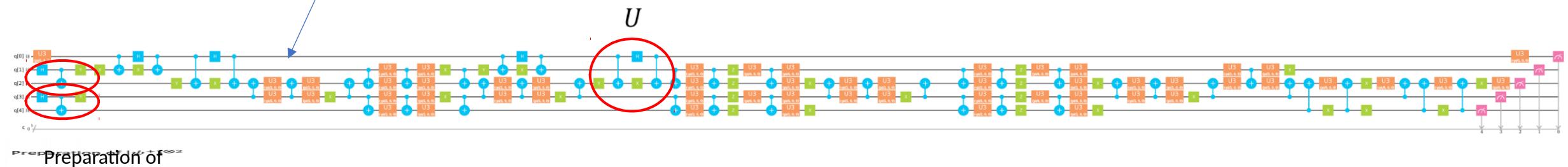


Largest circuit depth allowed!

IBM QX4: Raven

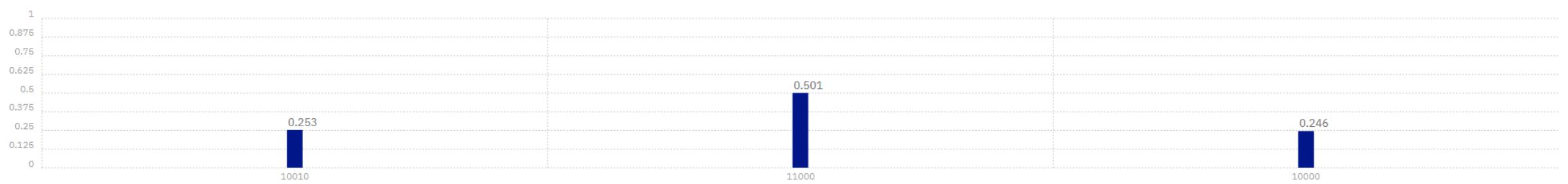


Target system



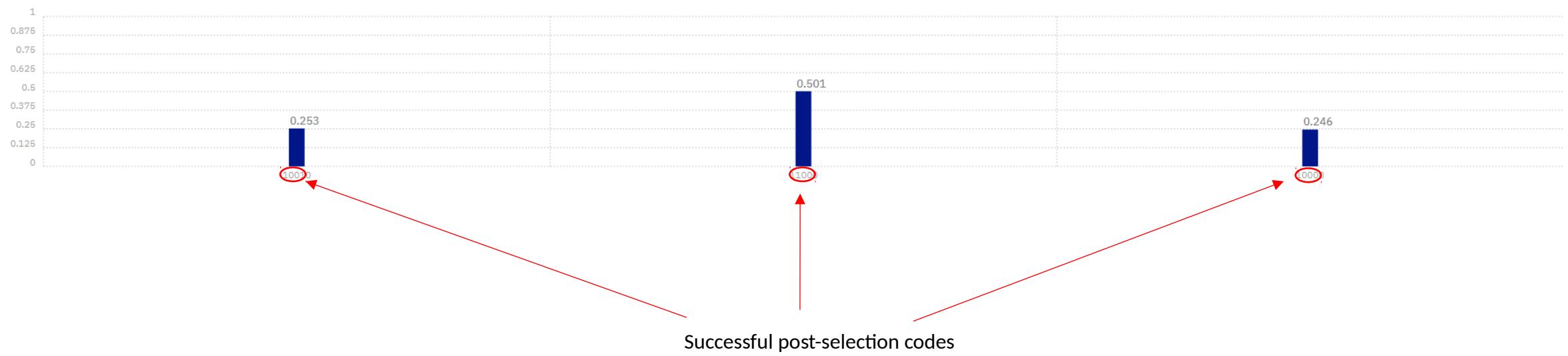
IBM QX4: Raven

Theoretical simulation



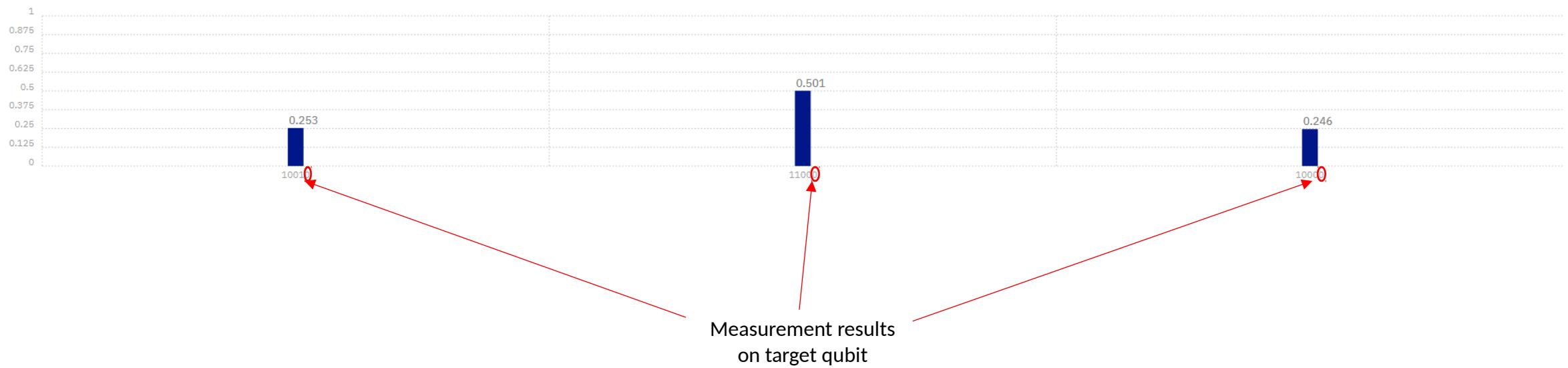
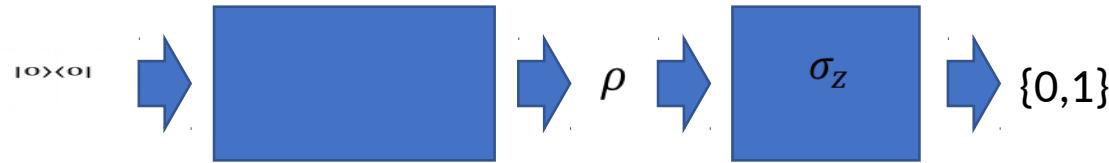
IBM QX4: Raven

Theoretical simulation



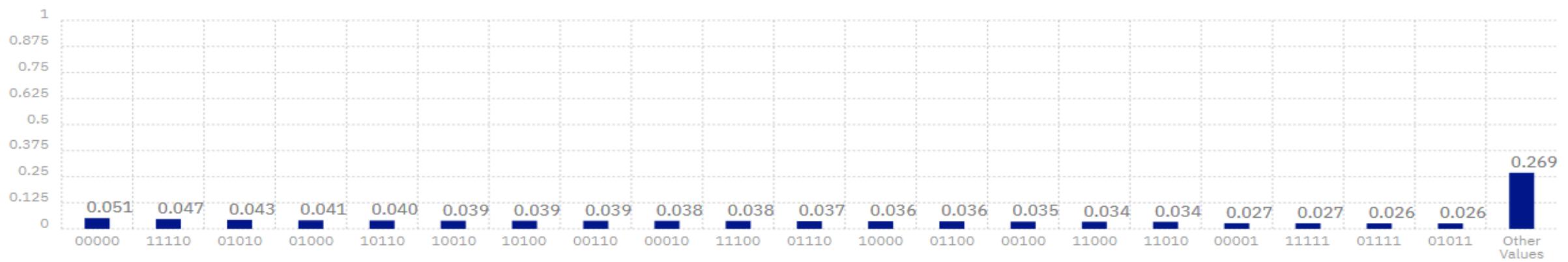
IBM QX4: Raven

Theoretical simulation

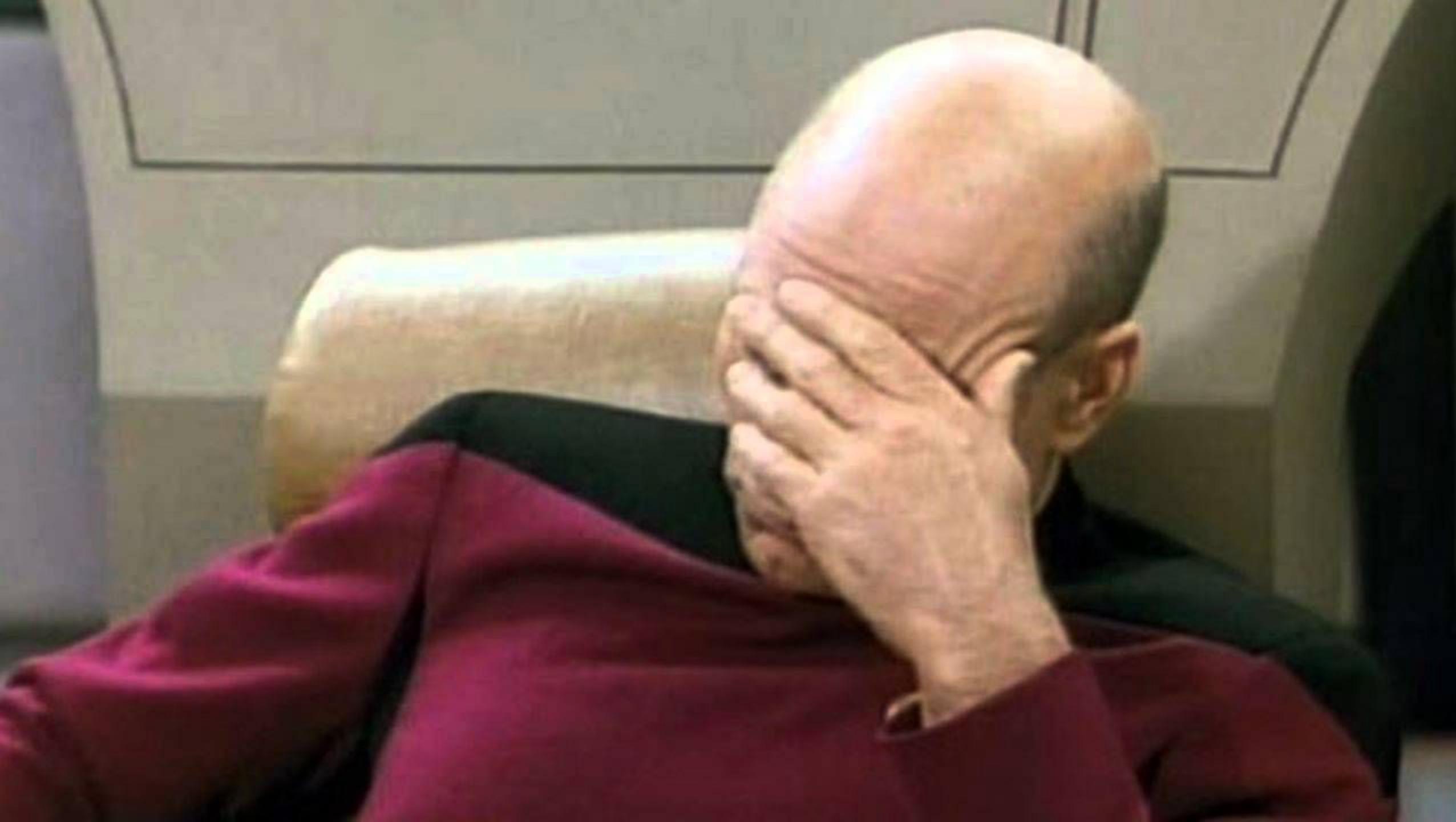


IBM QX4: Raven

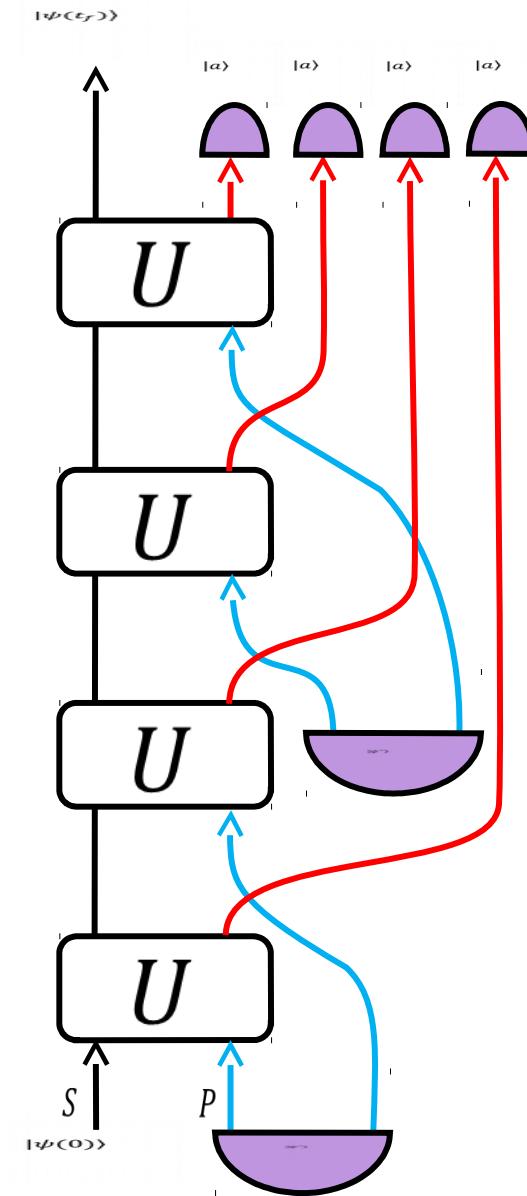
Crude reality

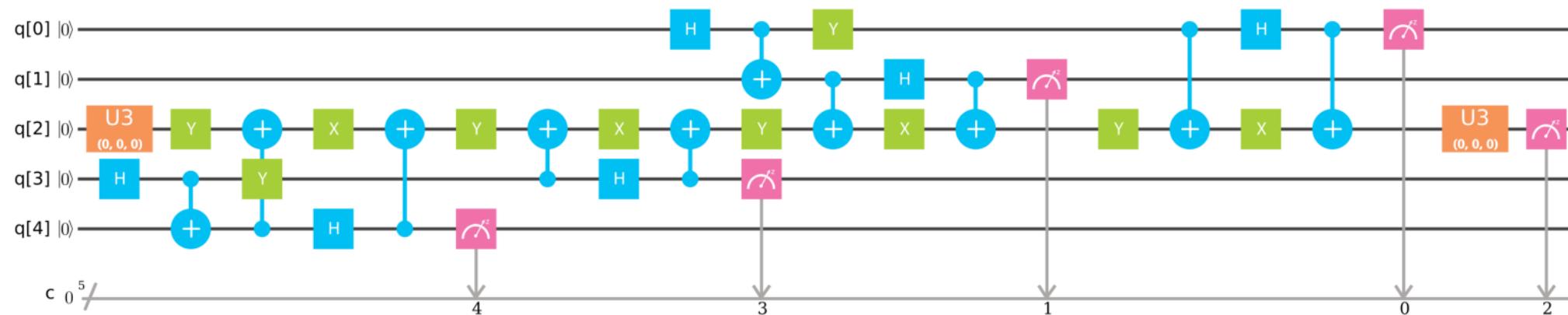
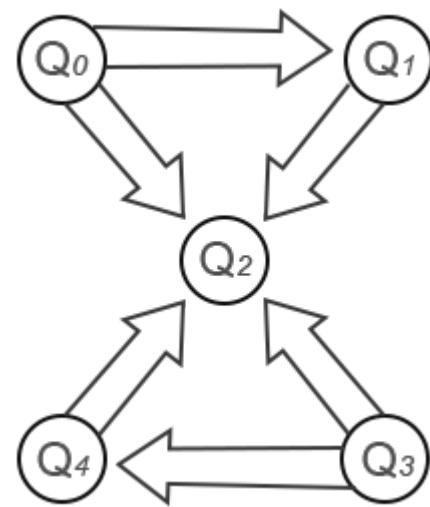


IBM QX4: Raven

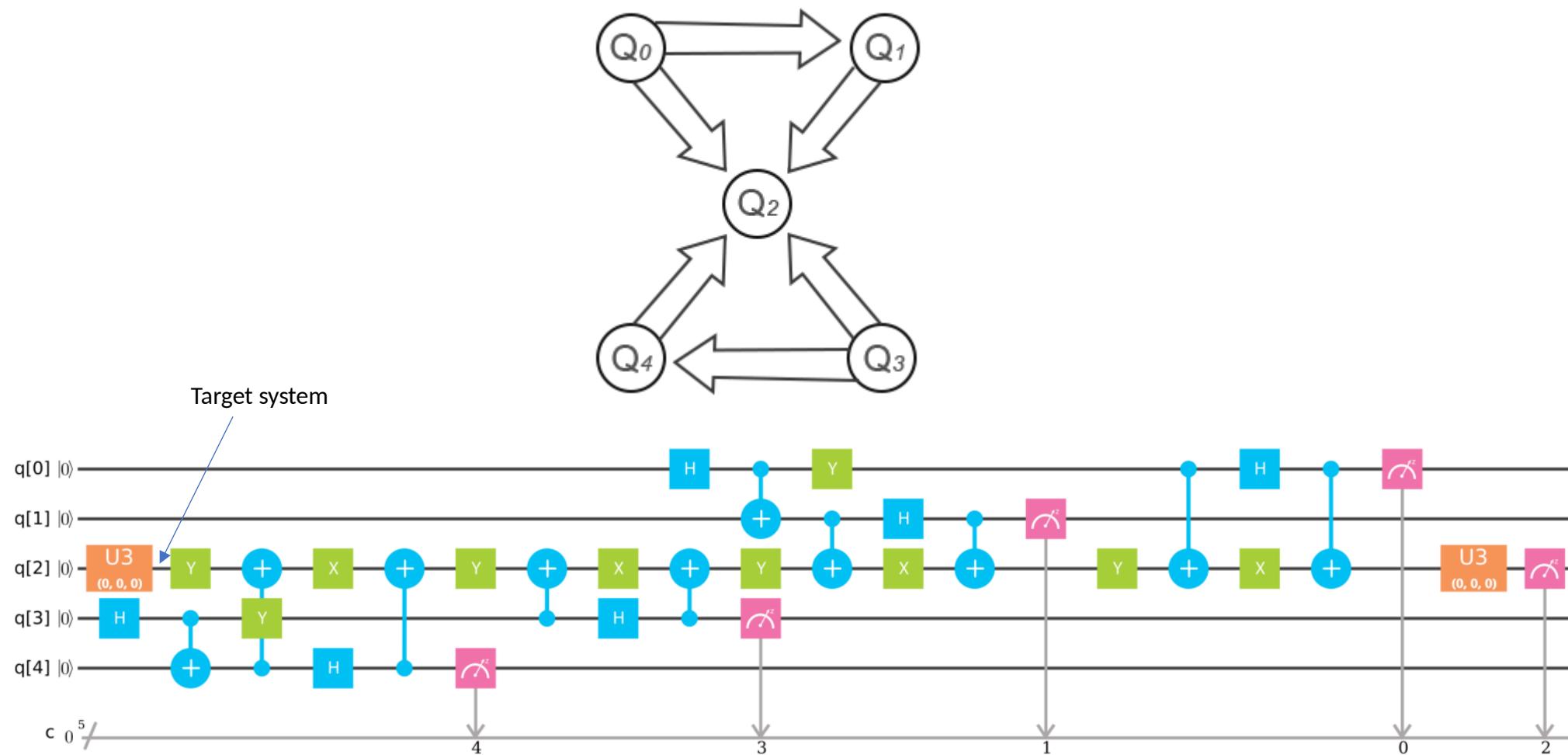


Experimental implementation: Prototype II



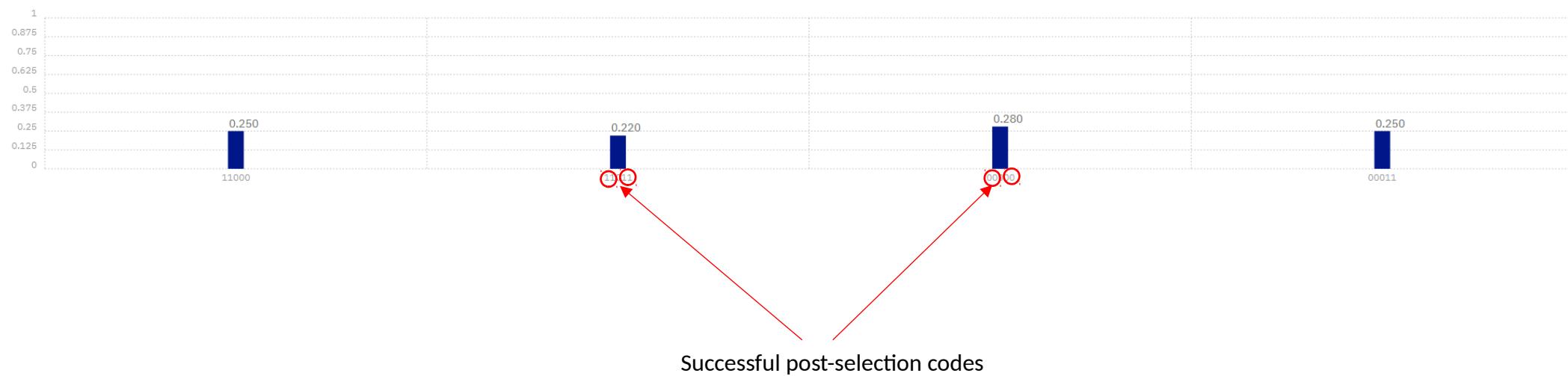


IBM QX2: Sparrow



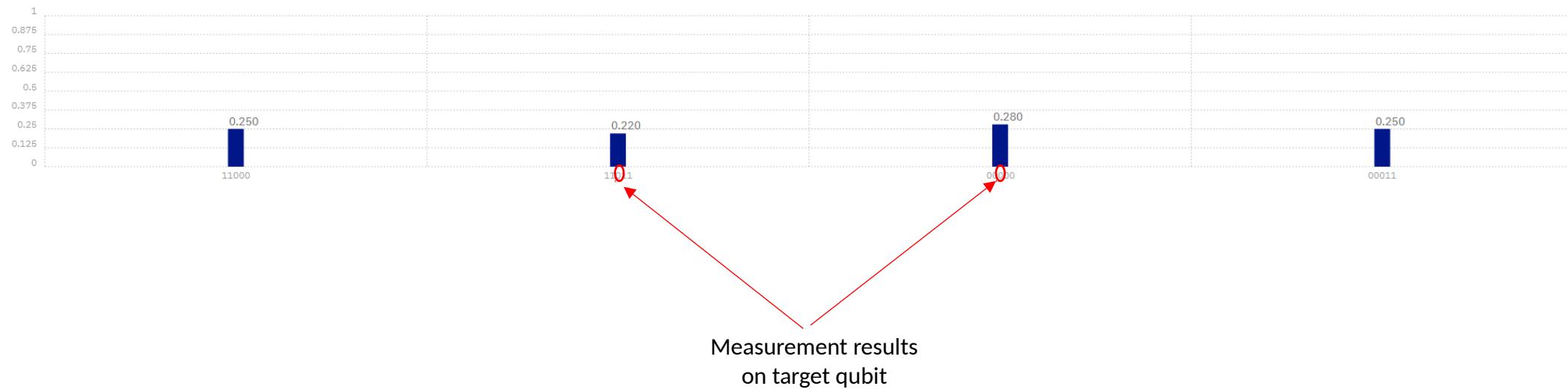
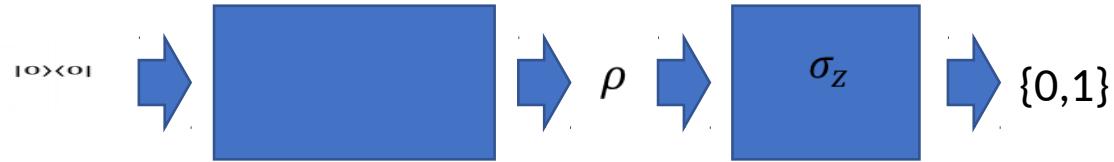
IBM QX2: Sparrow

Theoretical simulation



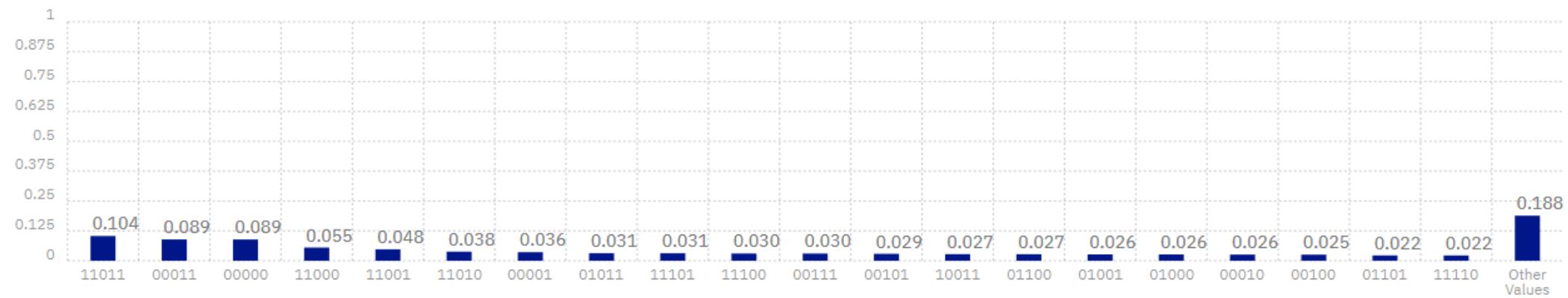
IBM QX2: Sparrow

Theoretical simulation



IBM QX2: Sparrow

Crude reality



IBM QX2: Sparrow

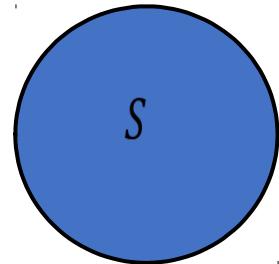


Conclusion

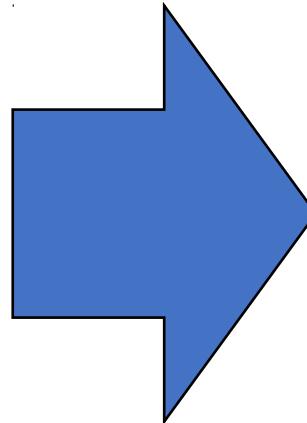
**Do there exist protocols with average probability of success (with prior dU) arbitrarily close to 1?
prior γ arbitrarily close to 1?**

Can we shorten resetting protocols?

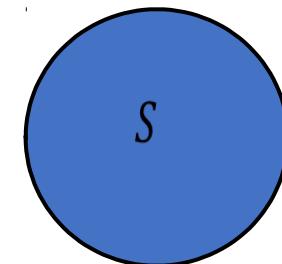
$$|\psi(T)\rangle = e^{-iH_0T}|\psi(0)\rangle$$



$$t = T$$



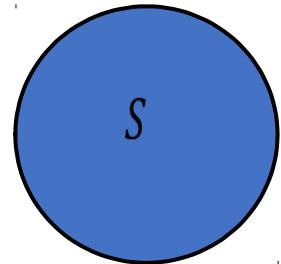
$$|\psi(0)\rangle$$



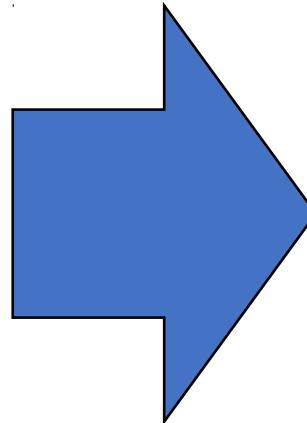
$$t = T + \Delta$$

Can we shorten resetting protocols?

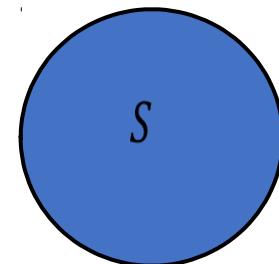
$$|\psi(T)\rangle = e^{-iH_0T}|\psi(0)\rangle$$



$$t = T$$



$$|\psi(0)\rangle$$

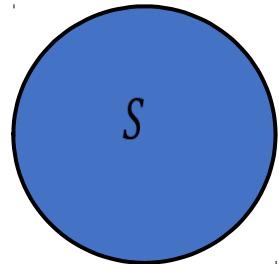


$$t = T + \Delta$$

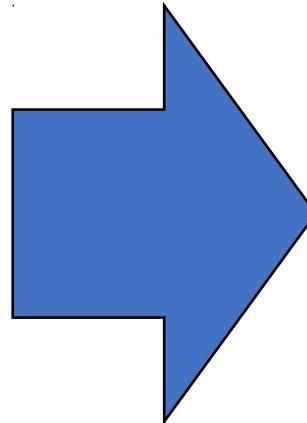
$$\Delta \geq 3T$$

Can we shorten resetting protocols?

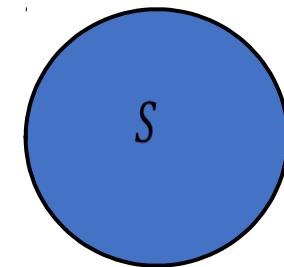
$$|\psi(T)\rangle = e^{-iH_0T}|\psi(0)\rangle$$



$$t = T$$



$$|\psi(0)\rangle$$

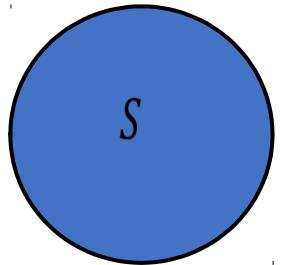


$$t = T + \Delta$$



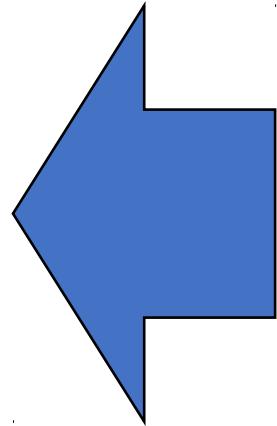
Can we fast-forward?

$$|\psi(T)\rangle = e^{-iH_0T}|\psi(0)\rangle$$

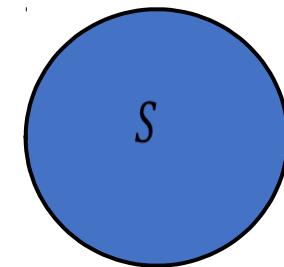


$$t = \Delta$$

$$\Delta \lessdot T ?$$

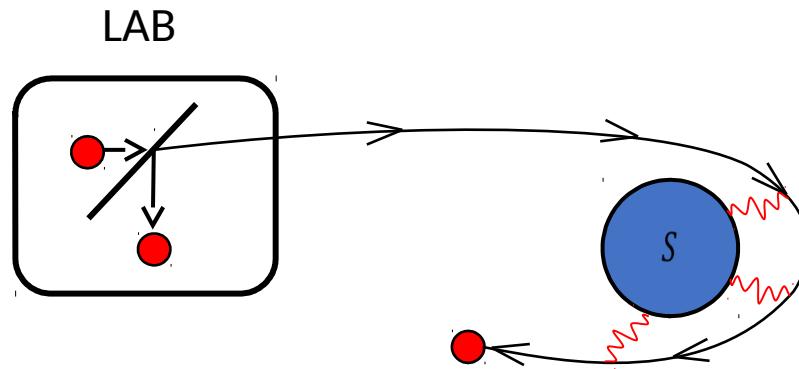


$$|\psi(0)\rangle$$



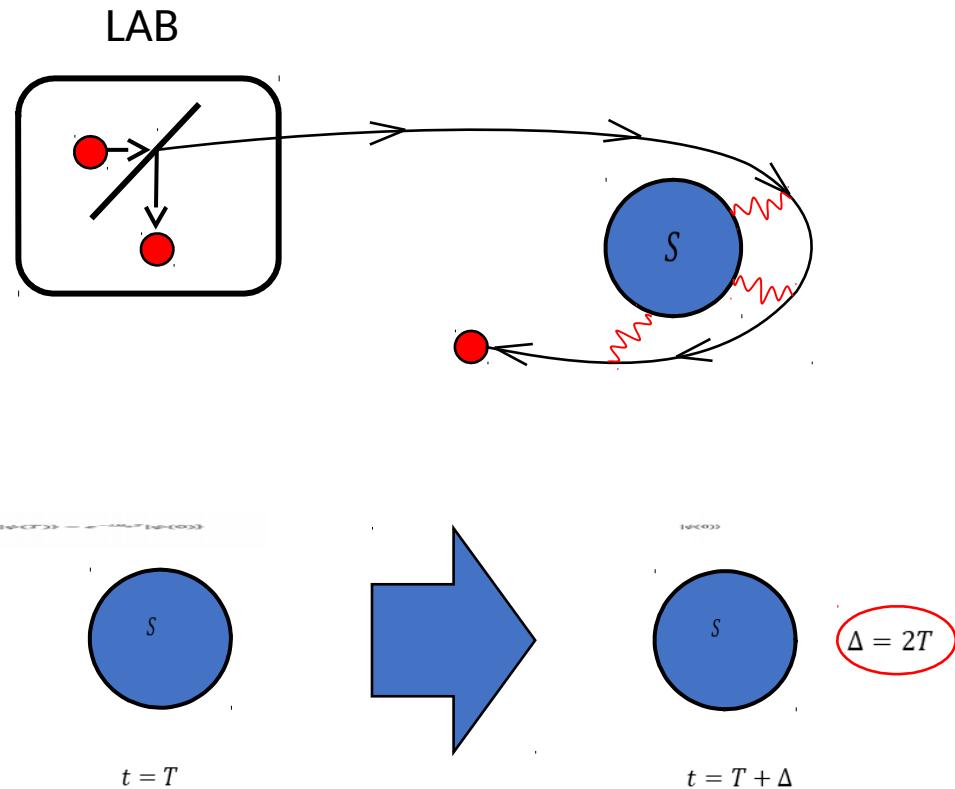
$$t = 0$$

Can we shorten resetting protocols?/Can we fast-forward?



Hint: use which-path superpositions.

Can we shorten resetting protocols?/Can we fast-forward?



MN, work in progress.

Simple experimental implementation?



Western Auto
World's Leading Auto Accessory

Auto
Sports

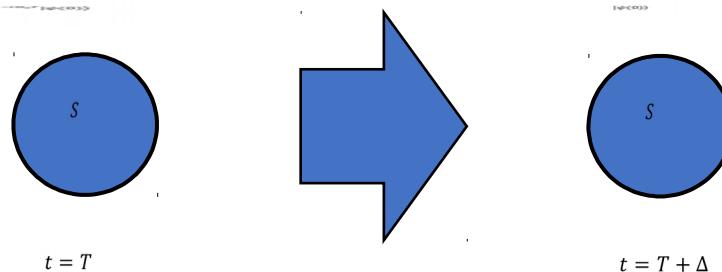
TO BE
CONCLUDED... ➤



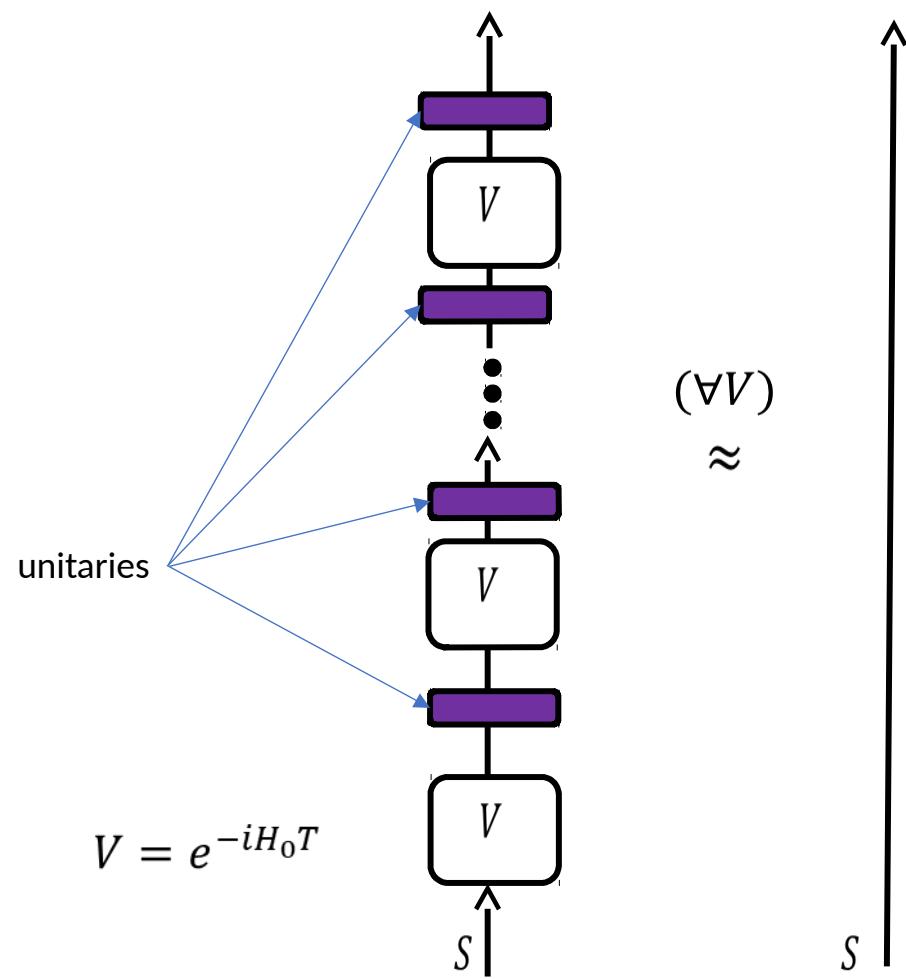
TOWN

HOLLYWOOD

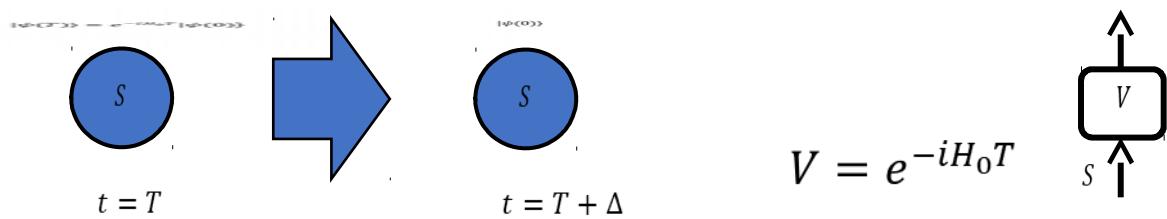
Refocusing



We ignore H but any operation O is allowed

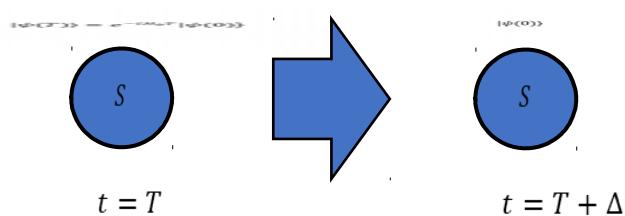


Refocusing (obvious solution)

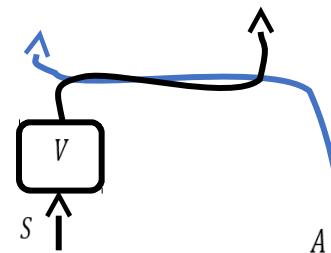


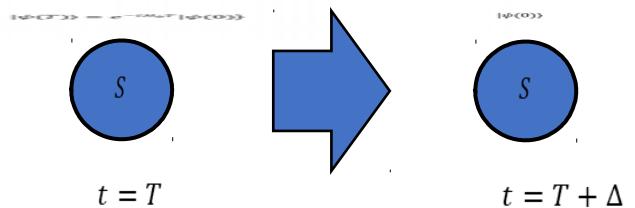
$$V = e^{-iH_0T}$$

Refocusing (obvious solution)



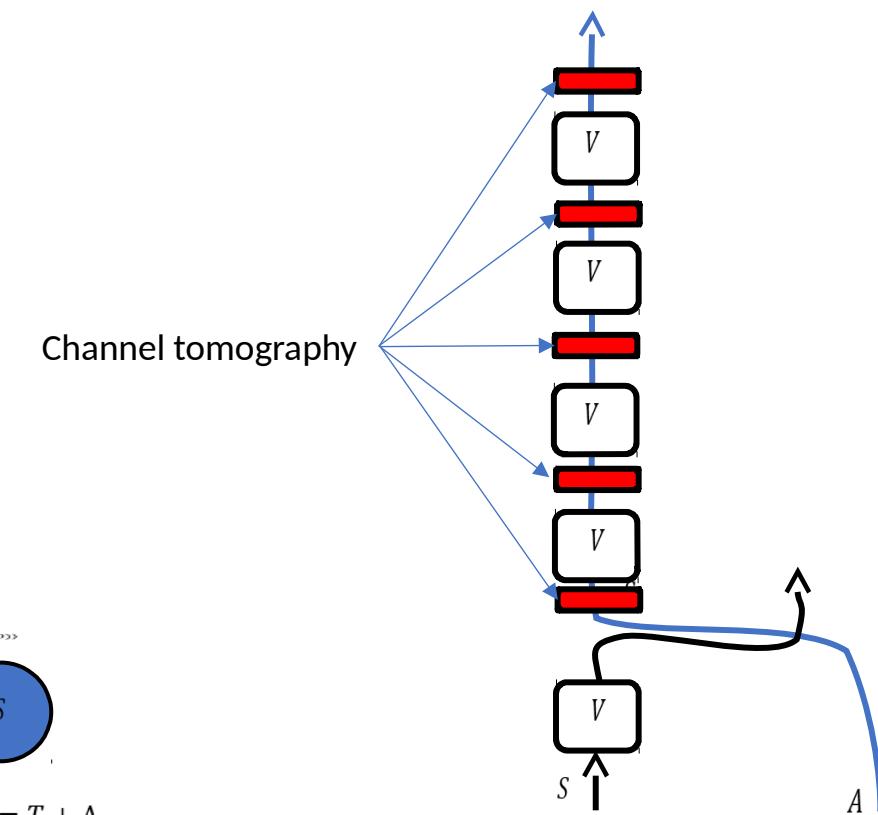
$$V = e^{-iH_0T}$$

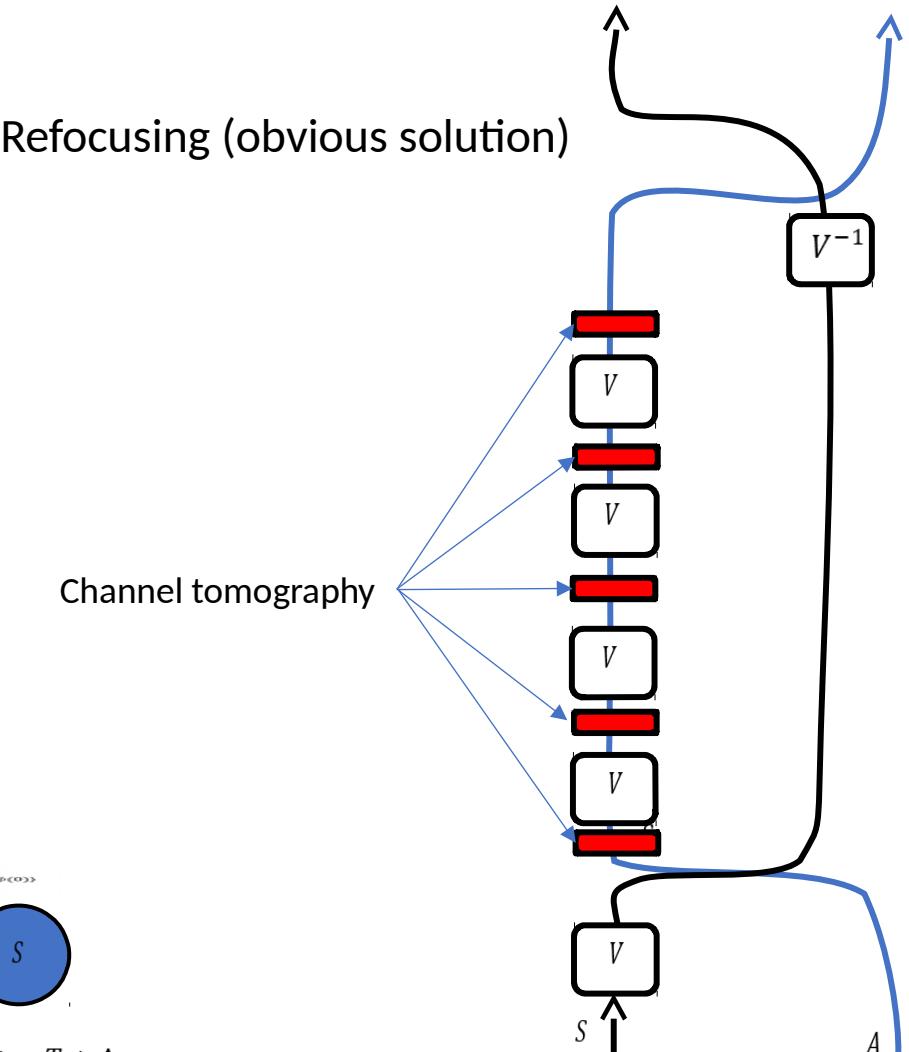
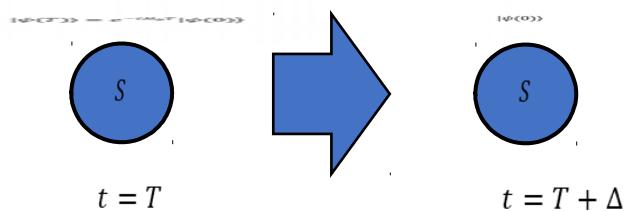


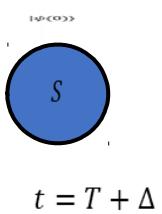
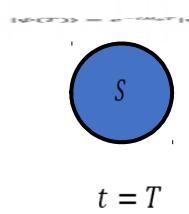


Refocusing (obvious solution)

Channel tomography

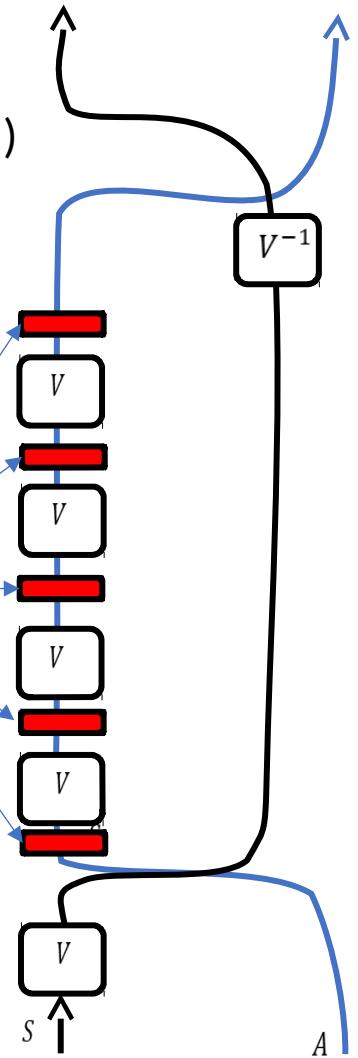






Refocusing (obvious solution)

Channel tomography



$$(\forall V) \approx$$

INPUTS
S
INPUTS