Composable security in relativistic quantum cryptography

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MOTIVATION

What is this talk about? (in 3 haikus)

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Start with resources,
Build a new one that's secure
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Exchanging quantum systems,
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Security from relativistic causality. E.g., Kent's 2012 bit commitment protocol

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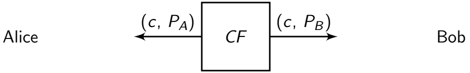
No model for this.
We propose one here and prove
What can, can't be done.

} **What we do**

Framework + new possibility, impossibility results

A simple relativistic coin flipping protocol



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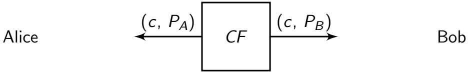

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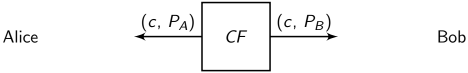

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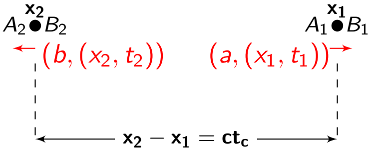

Output: $c = a \oplus b$ (in joint causal future) if $|t_1 - t_2| < \frac{t_c}{2}$

Composability issues

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(WO)MAN IN THE MIDDLE ATTACK

Anoushka

Bob

Charlie

David

Composability issues

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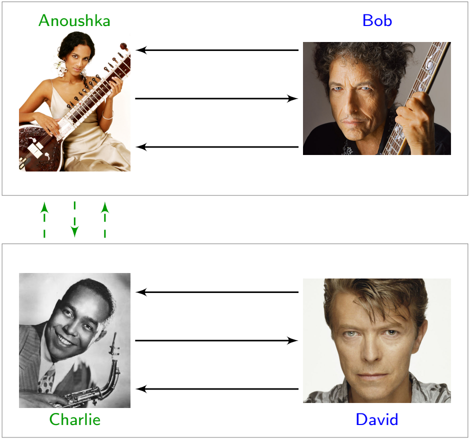

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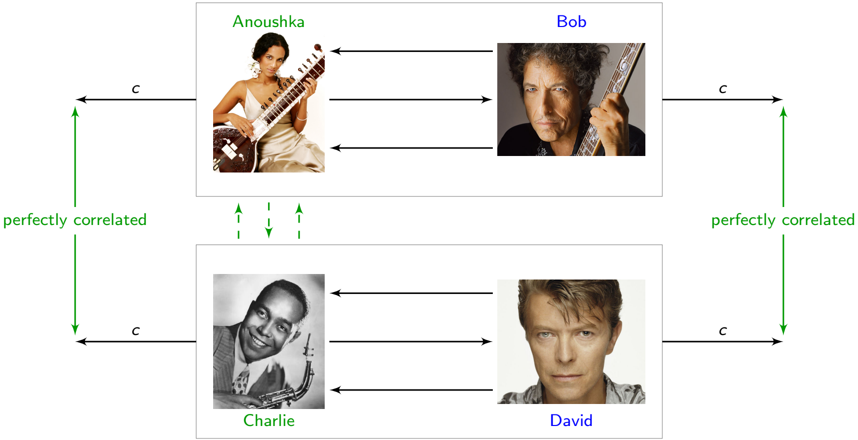

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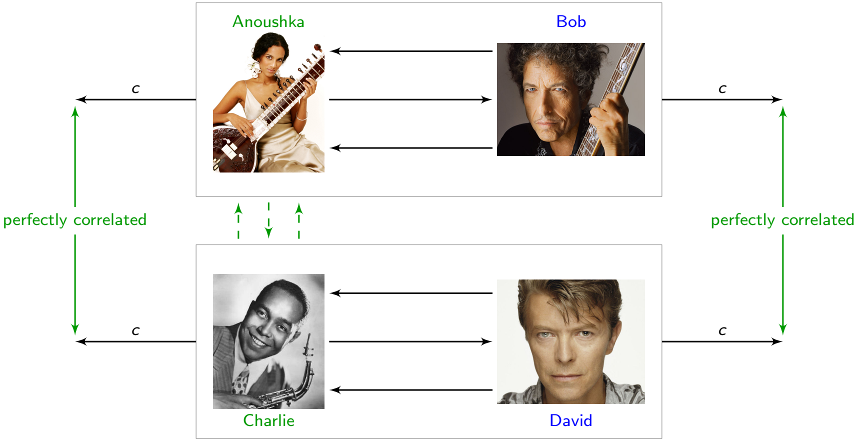

- MITM ⇒ pairs of parties cannot settle disputes independently i.e. $\mathcal{CF}$ not secure.
- Such an attack can be avoided if parties pre-share a bit commitment resource $\mathcal{BC}$.

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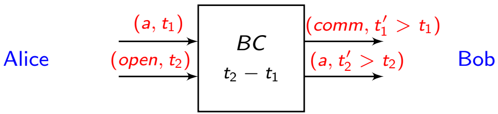

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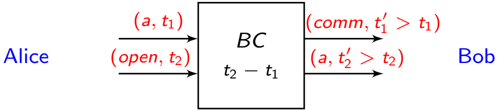
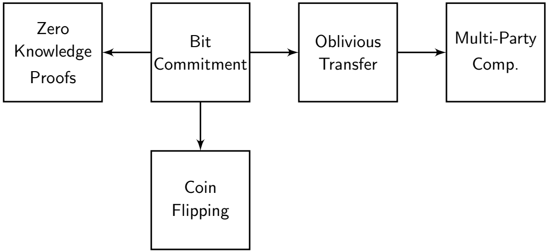


A Bit Commitment resource ($a \in \{0,1\}$).

- Arbitrarily long commitments.
- Committer can choose when to open or not to open at all.
- Relativistic protocols only allow for timed commitments of fixed duration. E.g., this makes protocols like Kent 2012 more like a "channel with delay".

State-of-the-art: BC solely through exchange of messages



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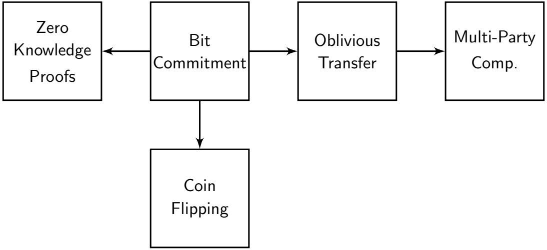

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  - **Stand-alone security:**No! quantum attack (MLC): Mayers, Lo, Chau 1996-1997.
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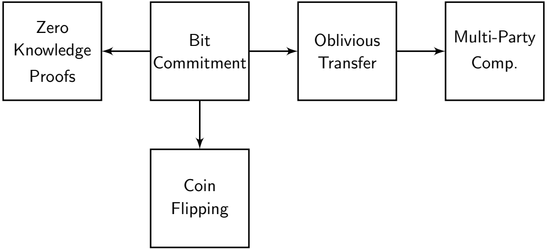

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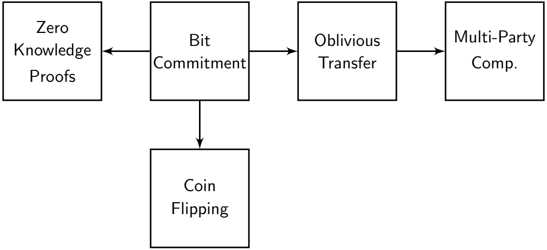

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No general framework for modelling composable security of relativistic protocols against classical, quantum, non-signalling adversaries.

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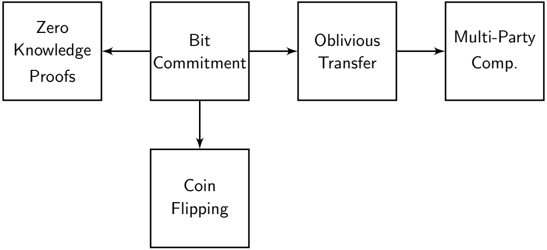

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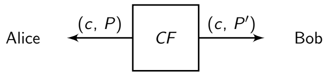
Our Work: Framework+new possibility/impossibility results.

THE FRAMEWORK

Resources (Abstract Cryptography[1])

- A resource is a system with interfaces, one for each player Alice and Bob providing them with certain controls.
- The resources available to the players are given by a tuple $\mathcal{R} = \{R, R_A, R_B\}$, defined by three resources: R when both parties are honest and R_i when party $i \in \{A, B\}$ is dishonest.

Example: coin flipping



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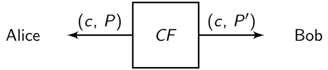

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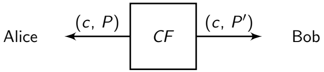

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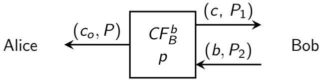
$$c_o = \begin{cases} b & \text{with prob. p} \\ c & \text{with prob. (1-p)} \end{cases}$$



$P_1 \prec P_2 \prec P$

(b) **A p biased resource:** The dishonest player can bias the value of honest player's output c_o towards a chosen bit b.

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Distance between resources: distinguishing advantage

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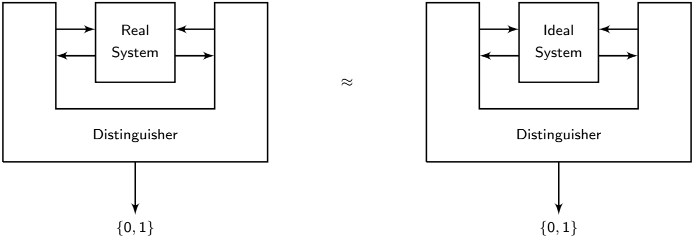
- Security is defined in terms of the indistinguishability of real systems from the corresponding ideal systems.

- $\mathcal{R} \approx_\epsilon \mathcal{S}$ for a class of distinguishers $\mathbb{D}$ if any distinguisher $\mathcal{D} \in \mathbb{D}$ when given black-box access to either one of the resources can distinguish between the two (by outputting 0 or 1) with a maximum probability of $(\epsilon + 1)/2$.



Security

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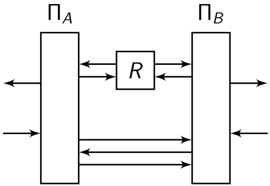

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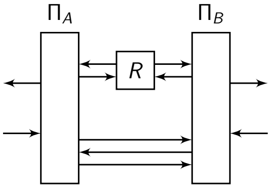

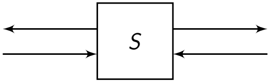

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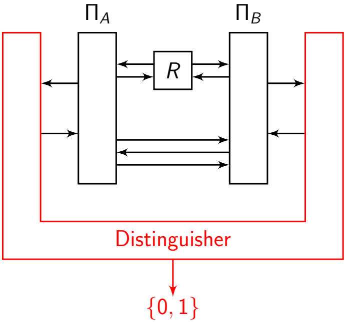

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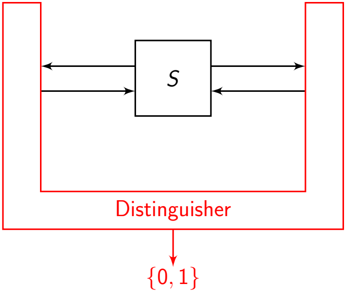

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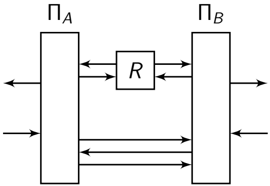

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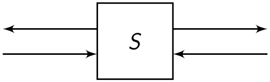

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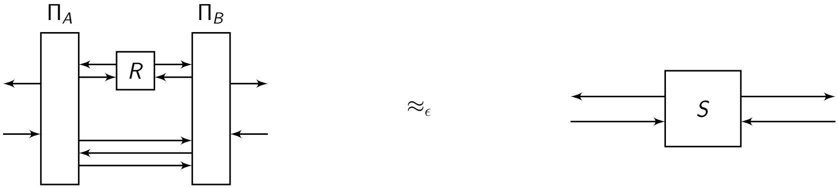

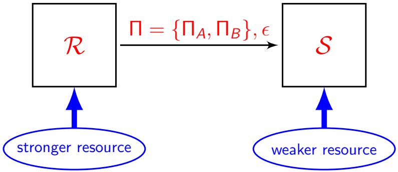

Causality (Causal Boxes[2])

Each system (resource, protocol, distinguisher etc.) is modelled as a **causal box**.

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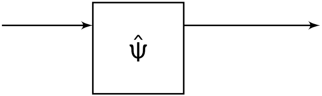

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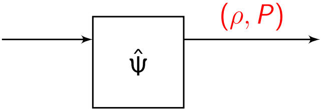

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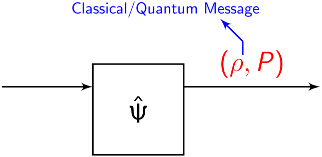

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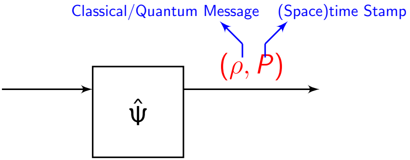

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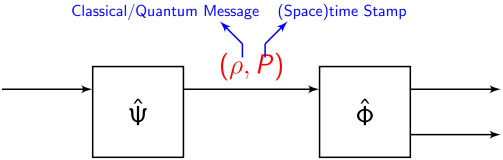

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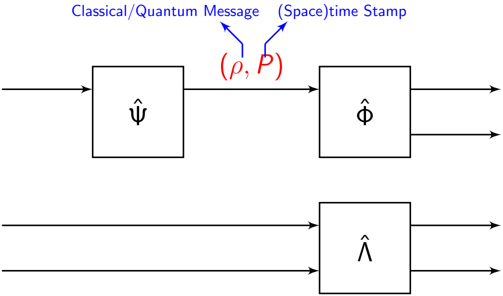

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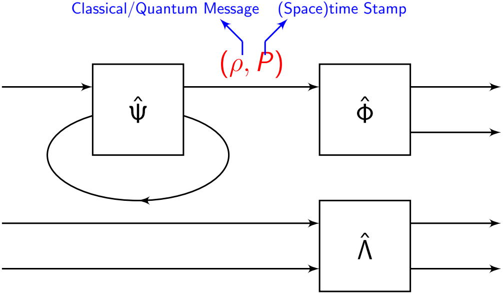

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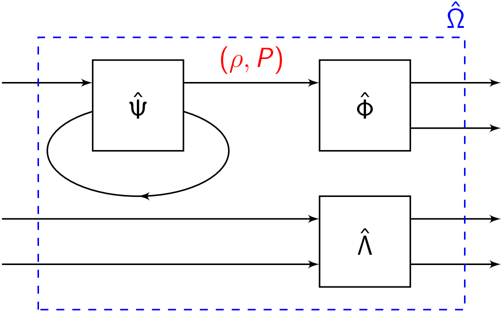

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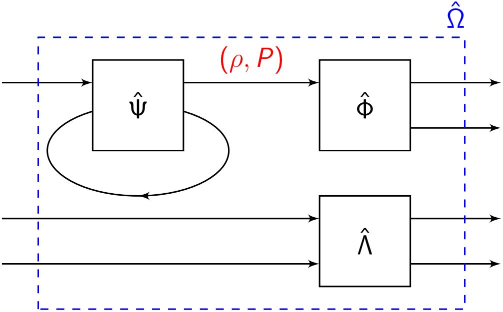

Composition: Arbitrary composition of CBs is a new CB, irrespecitve of order of composition.

Causality: An output of a system can only depend on inputs produced in its causal past.

Can model messages sent in superpositions of orders in space-time.

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New resource: channel with delay ($\mathcal{CD}$)

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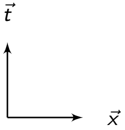

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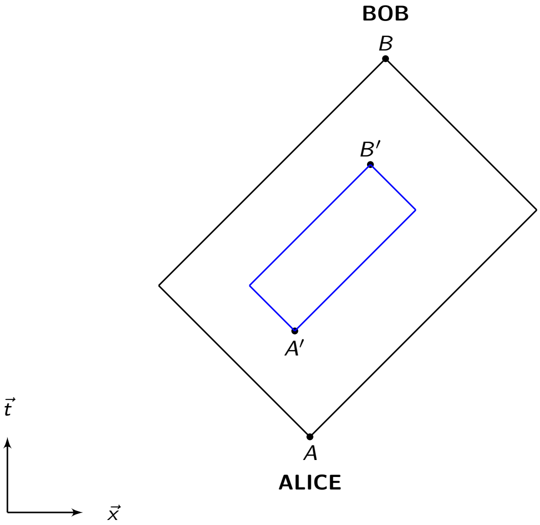

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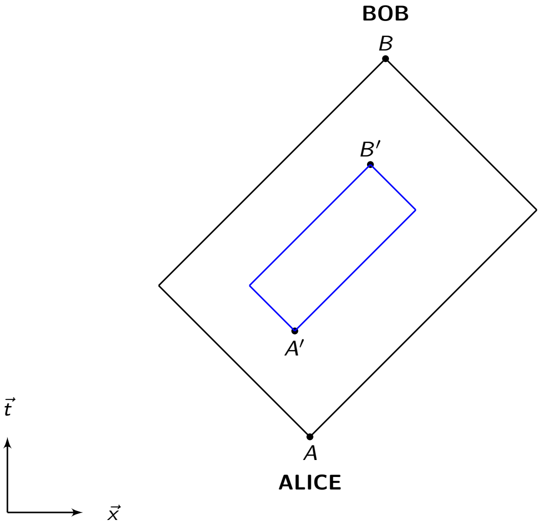

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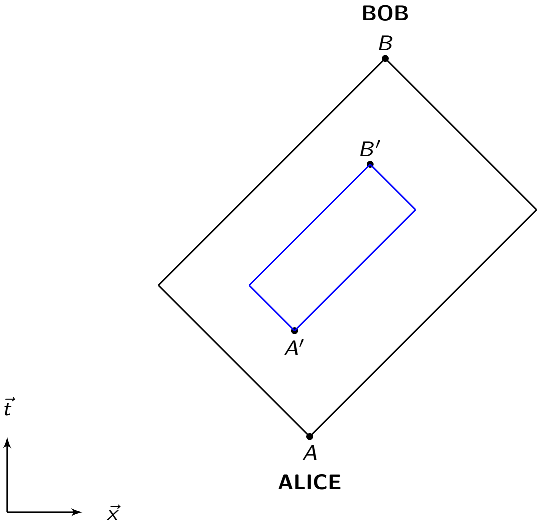

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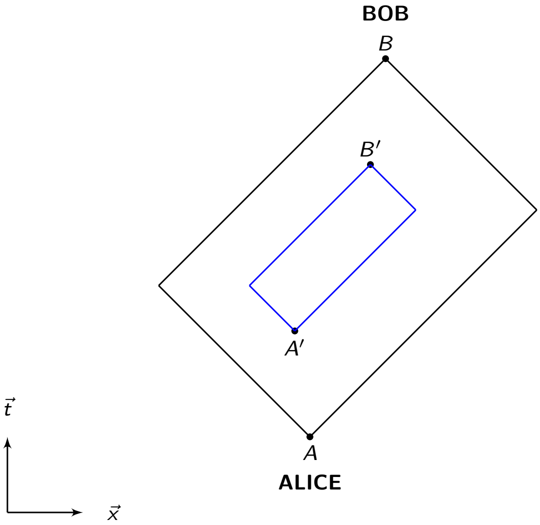

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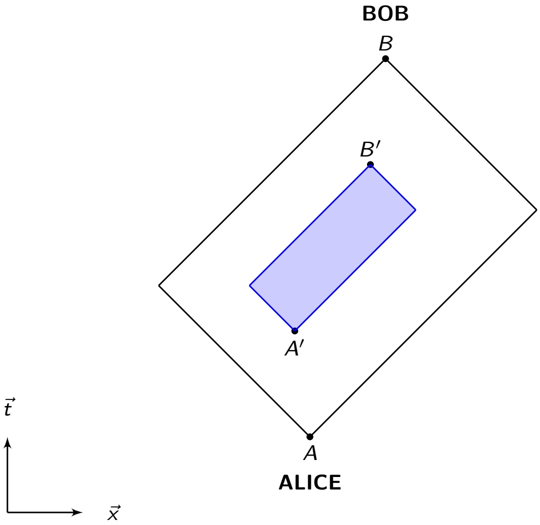
- CD_B**:** Alice inputs c/q-bit at A, Bob receives the same bit at $B' \prec B$

New resource: channel with delay ($\mathcal{CD}$)



- $\mathcal{CD} = \{CD, CD_A, CD_B\}$ is characterised by the 4 space-time points $A \prec A' \prec B' \prec B$

- CD**:** Alice inputs c/q-bit at A , Bob receives the same bit at B
- CD_A**:** Alice inputs c/q-bit at $A' \succ A$, Bob receives the same bit at B
- CD_B**:** Alice inputs c/q-bit at A, Bob receives the same bit at $B' \prec B$

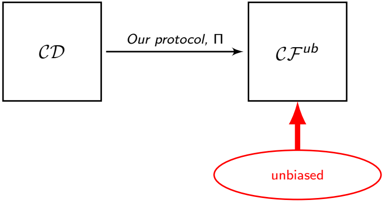
trusted region: region within which neither dishonest party can access the bit

RESULTS

Results: Constructibility of $\mathcal{CF}$ from $\mathcal{CD}$

Theorem 1

Given a classical Channel with Delay resource $\mathcal{CD}$, there exists a protocol $\Pi = \{\Pi_A, \Pi_B\}$ that perfectly constructs an unbiased Coin Flipping resource $\mathcal{CF}^{ub}$.



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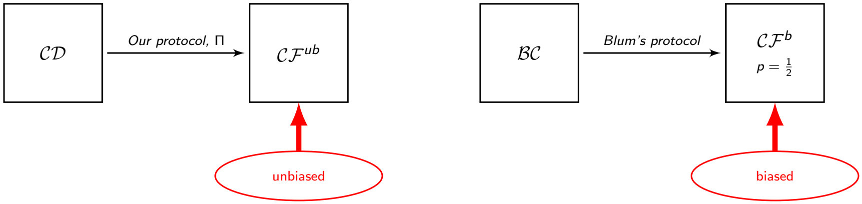

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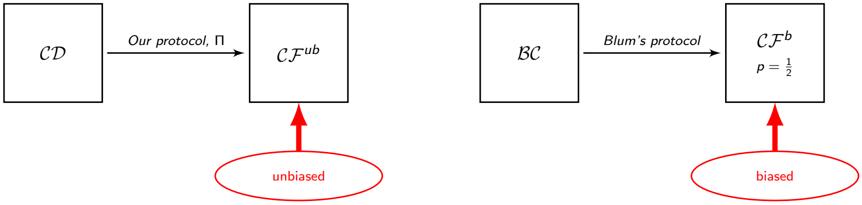

Π constructs a stronger resource as compared to Blum's protocol.

Secure against quantum and non-signalling adversaries

Results: Impossibility of relativistic bit commitment

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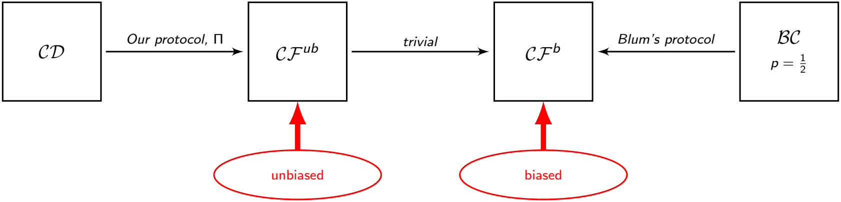
Theorem 2

It is impossible to construct, with $\epsilon < \frac{1}{6}(1-p)$, a p-biased Coin Flipping resource between two mutually distrusting parties solely through the exchange of messages through any relativistic or non-relativistic protocol, be it classical, quantum or non-signalling.



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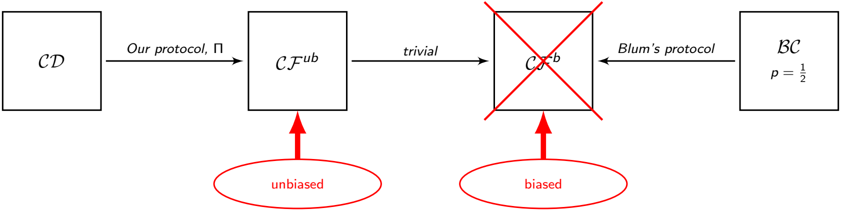

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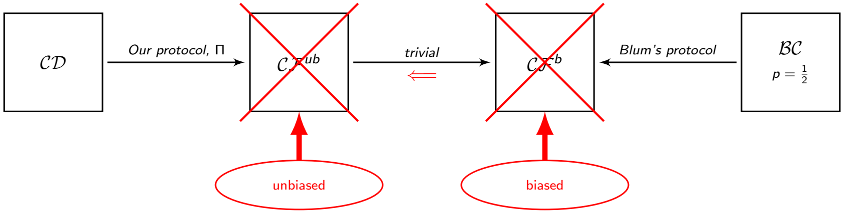

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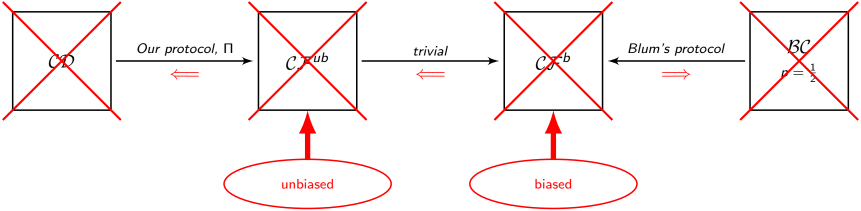
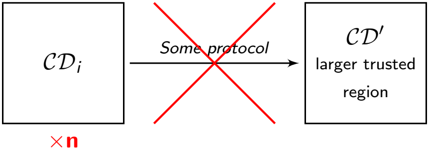


$\Rightarrow$ Existing protocols are not secure when composed, even in bounded/noisy storage models.

Results: Impossibility of "improving" a $\mathcal{CD}$

Theorem 3

Given n channel's $\mathcal{CD}^1$,...,$\mathcal{CD}^n$ between Alice and Bob, it is impossible to construct with $\epsilon \leq \frac{1}{8}$, a channel $\mathcal{CD}'$ between the two parties with a larger trusted region than that of all of the channels used.



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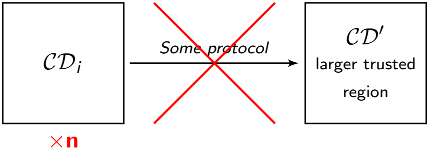

Impossible even if honest players send messages in a superposition of orders through the channels.

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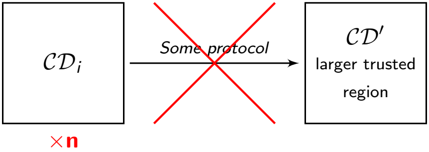

Impossible even if honest players send messages in a superposition of orders through the channels.

⇒ Cannot increase trusted region.

⇒ Cannot increase "effective commitment time" even with n channels.

Conclusions and Outlook

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- Physically motivated framework for studying spatio-temporal correlations and their applications to relativistic cryptography.
- Generalise to dynamical and indefinite causal structures, e.g., QM+GR.

Thank you for your attention!

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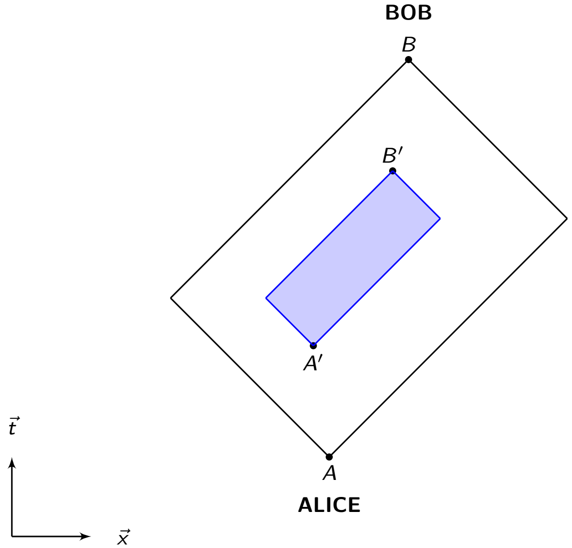
Additional Slides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ALICE
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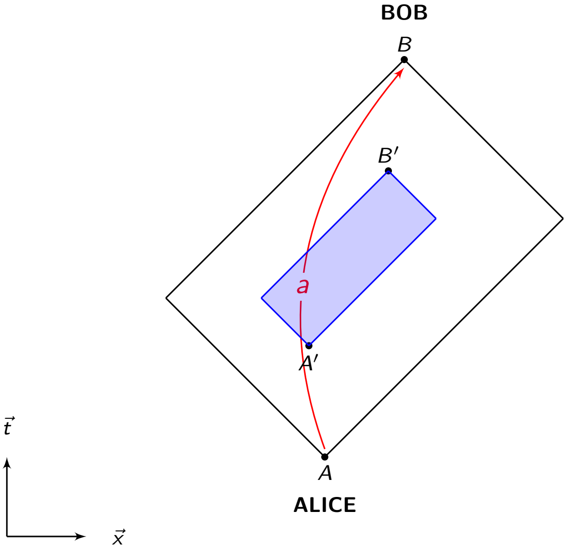
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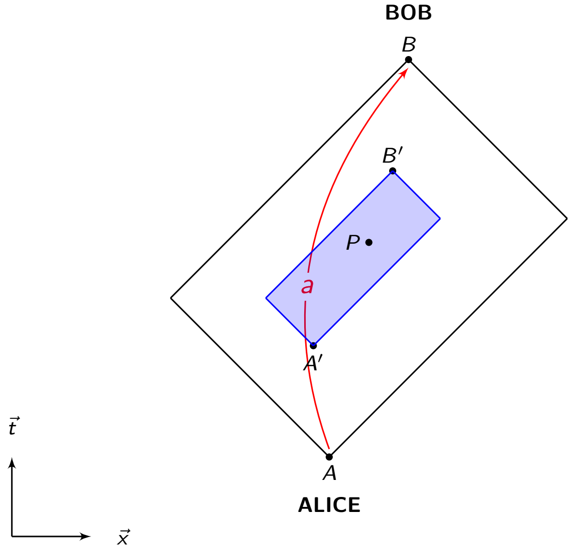
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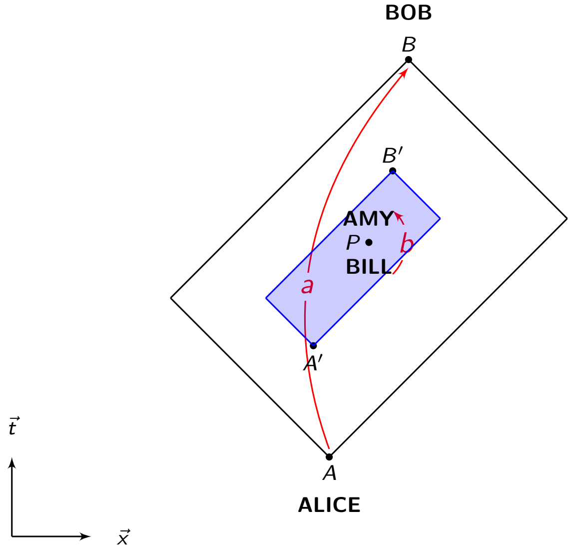
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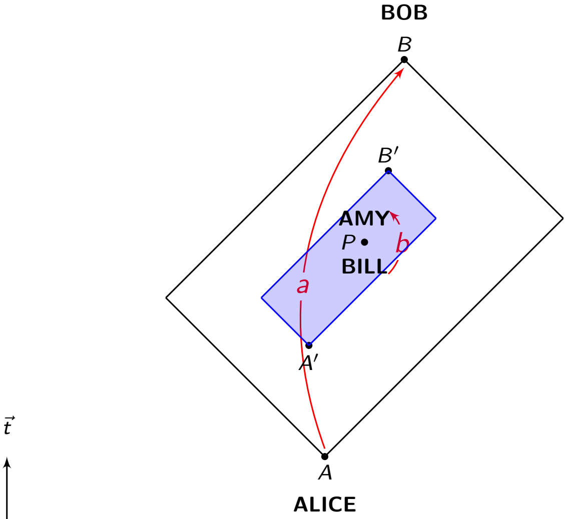
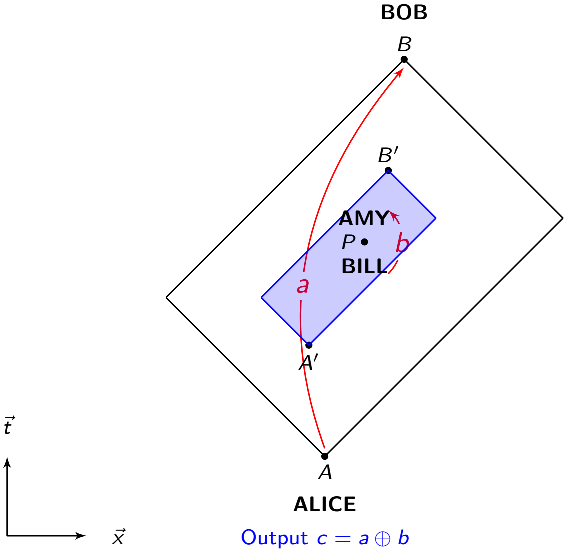


Output $c = a \oplus b$

Alice at $P_A \succ P$, Bob at $P_B \succ B$



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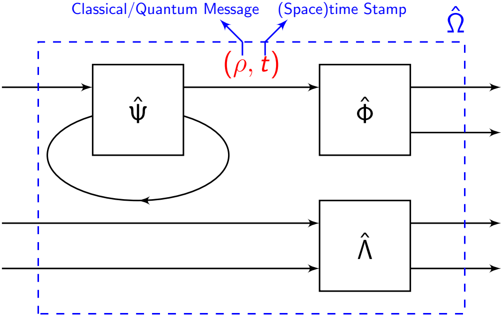
Secure against quantum and non-signalling adversaries

Discussion: indefinite causal structures



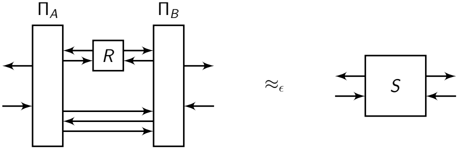
Causal Boxes (Portmann et. al. 2017)

- global and local order
- some indefinite causal structures (QS)
- quantum and NS (PR boxes)
- physically motivated

Process Matrices (Oreshkov et. al. 2012)

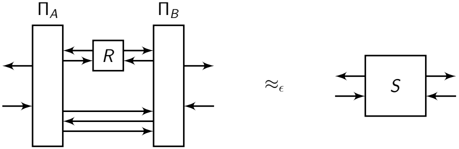
- no global, only local order
- QS+more general causal structures
- local quantum operations
- theoretical

Insights into properties of physical causal structures?



(a) $\Pi_A \, \mathrm{R} \, \Pi_B \approx_\epsilon \mathrm{S}$

- For every resource $\mathcal{R}$, three ideal functionalities are defined: R when both players are honest and R_i when player $i \in \{A, B\}$ is dishonest.



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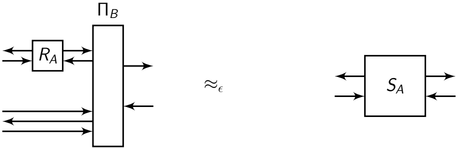

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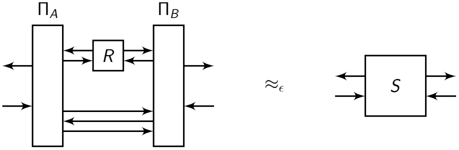

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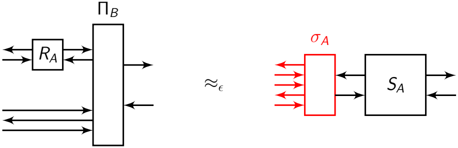

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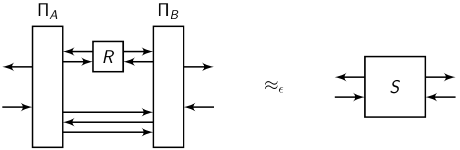

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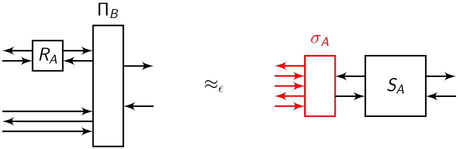

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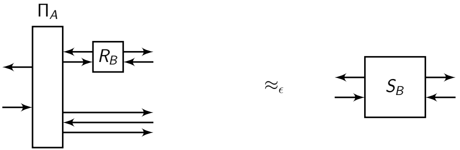

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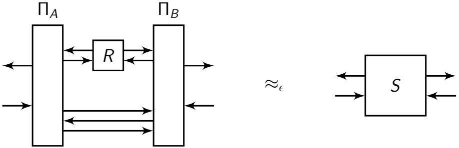

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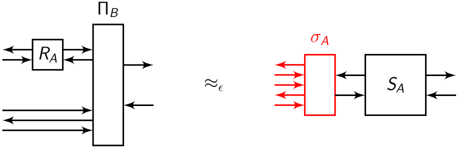

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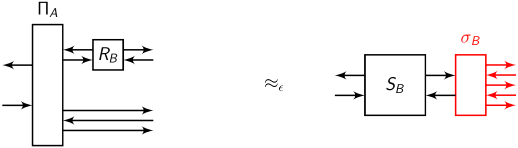

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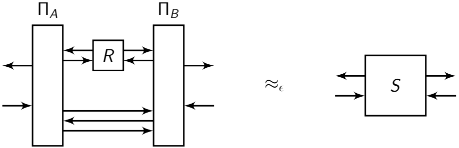

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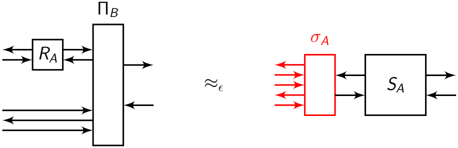

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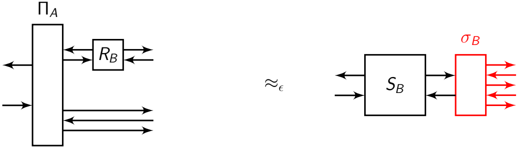

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- Composable Security: A protocol (Π_A, Π_B) constructs $\mathcal{S} = \{\mathrm{S}, \mathrm{S_A}, \mathrm{S_B}\}$ from $\mathcal{R} = \{\mathrm{R}, \mathrm{R_A}, \mathrm{R_B}\}$ securely within ϵ if $\exists \, \sigma_{\mathrm{A}}$ *and* σ_{B} for which the three conditions (a)-(c) hold.