# CONTEXTUALITY AS A PRECONDITION FOR ENTANGLEMENT

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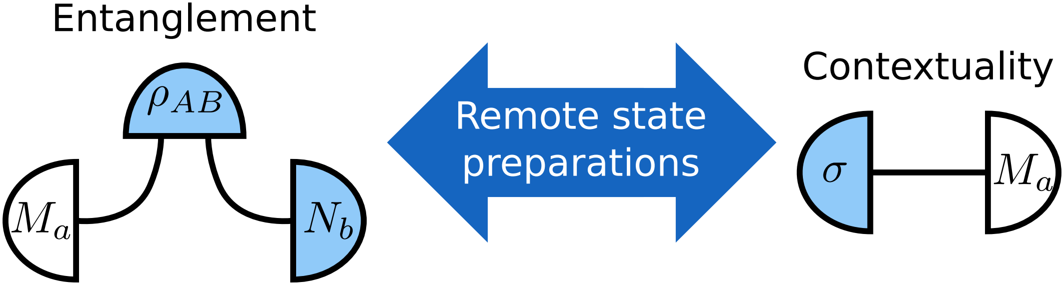

Entanglement
$\rho_{AB}$
$M_a$
$N_b$
Remote state preparations
Contextuality
$\sigma$
$M_a$

# Remote preparations

**Main idea**

1. Alice and Bob share $\rho_{AB}$
2. Bob measures $\{N_b\}$ and obtains outcome $b$.
3. Bob announces outcome $b$.

State of Alice's system:

$$\sigma_A \approx \mathrm{Tr}_B((\mathbb{1}_A \otimes N_b)\rho_{AB})$$

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Set of all remotely-preparable states:

$$\Lambda_A(\rho_{AB}) = \{\sigma_A \in \mathcal{D}(\mathcal{H}_A) : E_B \geq 0, \sigma_A = \mathrm{Tr}_B((\mathbb{1}_A \otimes E_B)\rho_{AB})\}$$

# Contextuality

$K$ - set of preparable states: $K \subset \mathcal{D}(\mathcal{H})$

**P&M contextuality**

$K$ has preparation & measurement noncontextual model if:

$$\mathrm{Tr}(\rho M_a) = \sum_{\lambda} \mathrm{Tr}(\rho N_\lambda)\,\mathrm{Tr}(\omega_\lambda M_a)$$

where

$$\mathrm{Tr}(\rho N_\lambda) \geq 0, \quad \forall \rho \in K, \forall \lambda$$

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$\mathrm{Tr}(\rho N_\lambda)$ - classical state replacing $\rho$
$\mathrm{Tr}(\omega_\lambda M_a)$ - classical response function replacing $M_a$

# Results

$\Lambda_A(\rho_{AB})$ has P&M noncontextual model

$$\Downarrow$$

$\rho_{AB}$ is separable

$$\Downarrow$$

$\Lambda_A((1-\varepsilon)\rho_{AB}+\varepsilon\tau_{AB})$ has P&M noncontextual model
for almost all separable $\tau_{AB}$ and $\varepsilon \in (0, \delta(\tau_{AB}))$

# Robust remote preparations are necessary

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$$\not\Downarrow$$

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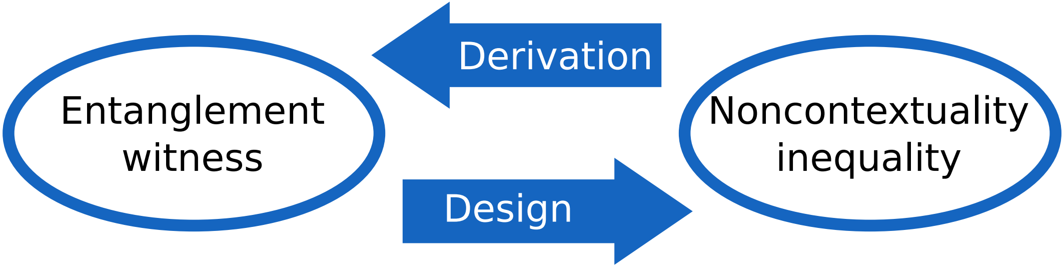
**SpongeBob has a counter-example**

$$\rho_{AB} = \frac{1}{4}(|0\rangle\langle 0| \otimes |00\rangle\langle 00| + |1\rangle\langle 1| \otimes |01\rangle\langle 01| + |+\rangle\langle +| \otimes |10\rangle\langle 10| + |-\rangle\langle -| \otimes |11\rangle\langle 11|)$$


Entanglement witness
Derivation
Design
Noncontextuality inequality

# New entanglement witness

**Proposition**

*Let $\rho_{AB}$ be a **separable quantum state**.*

*Let $E_{t,b}$ and $M_{t,b}$ be positive operators, $E_{t,b} \geq 0$ and $M_{t,b} \geq 0$, such that $E_* = \frac{1}{2}(E_{t,0} + E_{t,1})$, $\frac{1}{3}\sum_{t=1}^{3} M_{t,b} = \frac{\mathbb{1}_A}{2}$ and $\mathbb{1}_A = M_{t,0} + M_{t,1}$ for all $t \in \{1, 2, 3\}$ and $b \in \{0, 1\}$. Then*

$$\sum_{t=1}^{3} \sum_{b=0}^{1} \mathrm{Tr}[(M_{t,b} \otimes E_{t,b})\rho_{AB}] \leq 5\,\mathrm{Tr}[(\mathbb{1}_A \otimes E_*)\rho_{AB}].$$

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For specific choice we get:

$$\mathrm{Tr}((\sigma_x \otimes \sigma_x)\rho_{AB}) + \mathrm{Tr}((\sigma_z \otimes \sigma_z)\rho_{AB}) \leq \frac{4}{3}$$

# New contextuality inequality

**Proposition**

*Let* $K$ *be a set of allowed preparations. Let* $\sigma_* \in K$ *and let* $i \in \{1, 2\}$*, let* $\sigma_{i+}, \sigma_{i-}, \sigma_{i0} \in \mathrm{cone}(K)$ *be subnormalized preparations such that*

$$\sigma_{1+} + \sigma_{1-} + \sigma_{10} = \sigma_* = \sigma_{2+} + \sigma_{2-} + \sigma_{20}.$$

*Let* $A_i$ *be observables such that* $-\mathbb{1} \leq A_i \leq \mathbb{1}$ *for all* $i \in \{1, 2\}$*. If* ***there is a P&M contextual model for*** $K$*, then*

$$\mathrm{Tr}[(A_1 + A_2)(\sigma_{1+} - \sigma_{1-})] + \mathrm{Tr}[(A_1 - A_2)(\sigma_{2+} - \sigma_{2-})] \leq 2.$$

Alexander von Humboldt
Stiftung / Foundation

The speaker's attendance at this conference was sponsored by the Alexander von Humboldt Foundation.

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